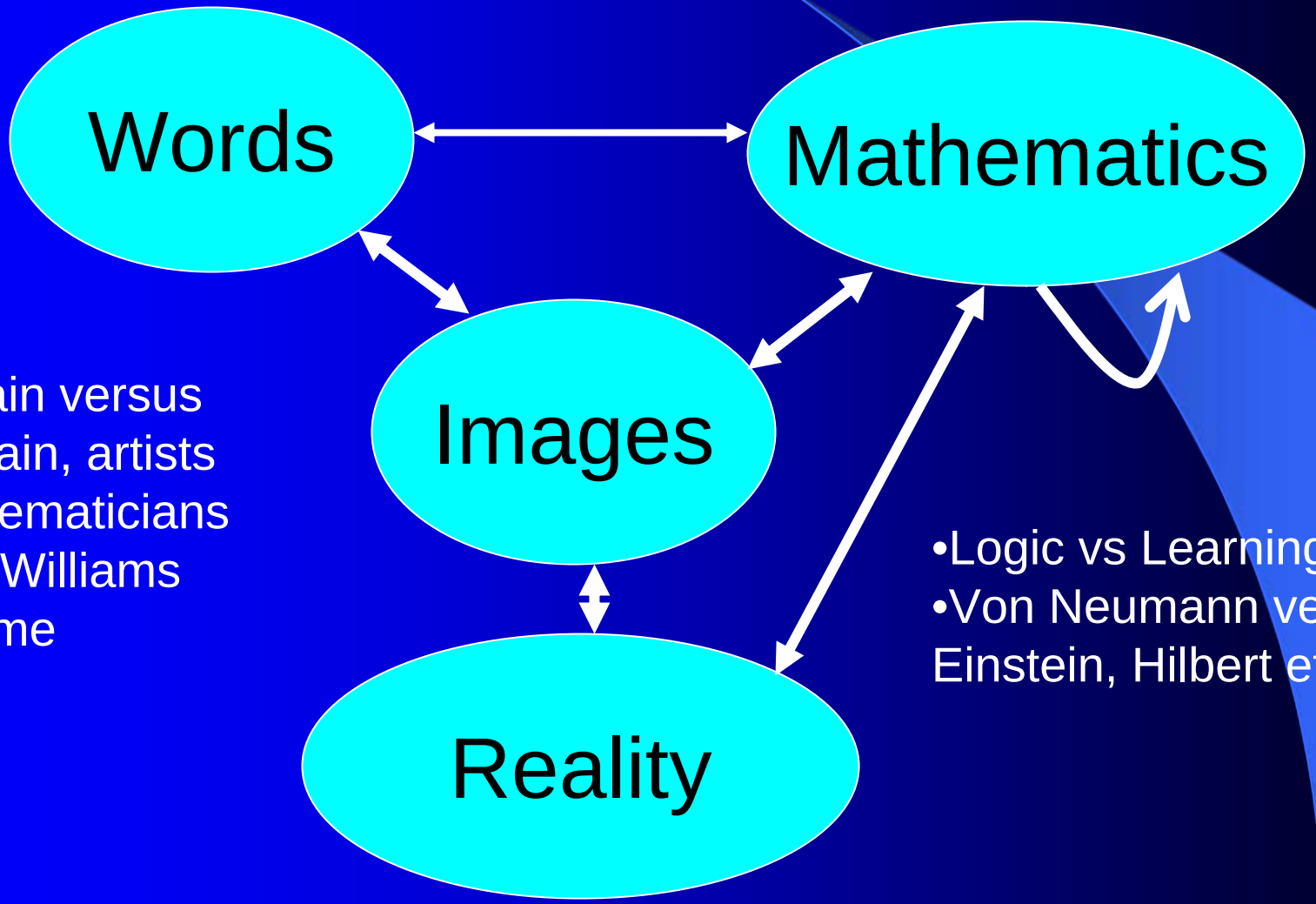
Mathematics and the Brain

Dr. Paul J. Werbos
National Science Foundation ($6b/yr..)

- General thoughts – your future, what is mathematics, link to investment opportunities, megachallenges
- Derivatives – the key to physics, to economics and the key to the brain and the mind
- Optimization – the concept and the neural designs
- Where to learn more – www.werbos.com

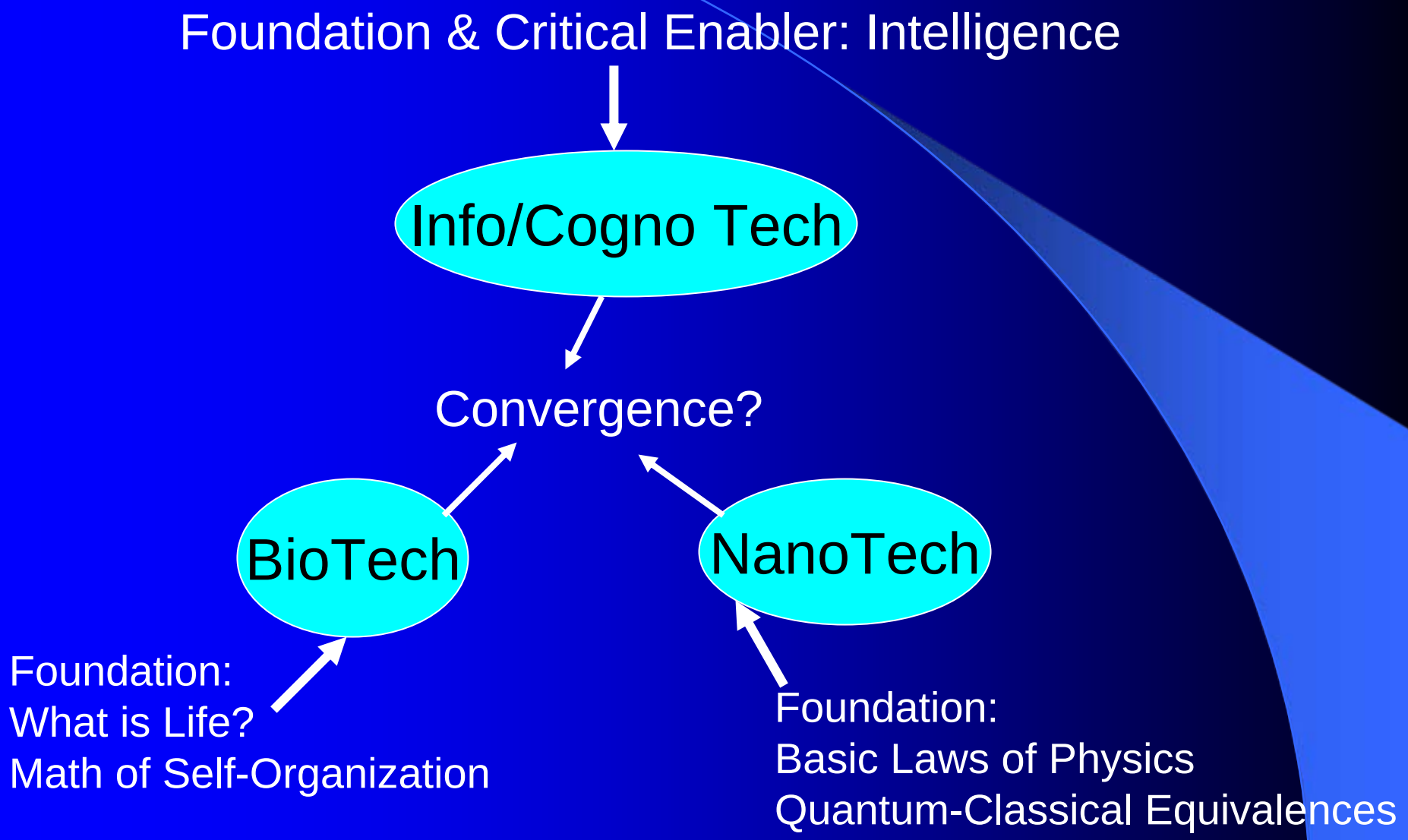
Mathematics Is: (1) a Language; and (2) a system of reasoning – "IF/THEN"



Left brain versus right brain, artists & mathematicians versus Williams syndrome

- Logic vs Learning
- Von Neumann versus Einstein, Hilbert etc.

Economic Frontiers Driven by Foundations

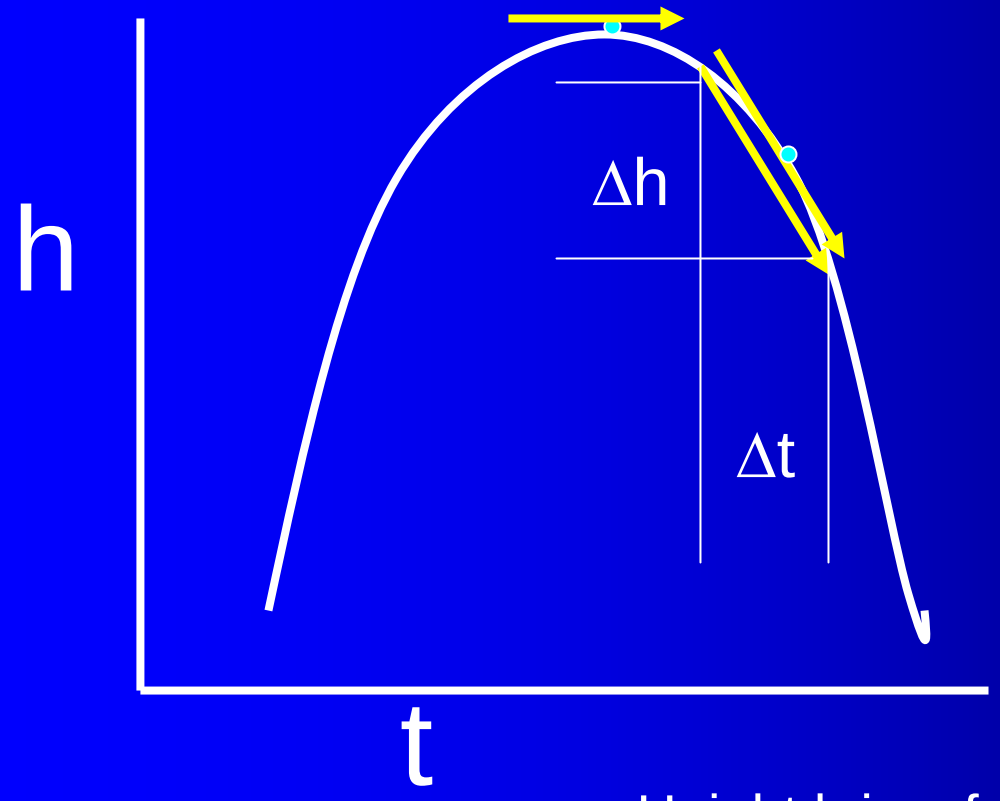


&: Converge in Foundations or Just Wires in Head?

6 MegaChallenges for the 21st Century

- Key Challenges To Basic Scientific Understanding:
  - What is Mind? (how to build/understand intelligence)
    - Basic Science of Mind: Up To the Highest Kind of General Intelligence We see in the Smallest Mouse
    - Middle Sci. Mind: from Mouse to "Sapient," the level of full use of symbolic reasoning integrated with meaning/empathy (human brain is "new", still halfway there in its evolution)
    - Higher Sci. of Mind: Principles of higher levels in intelligent systems design, like quantum, multimodular, soul
  - How does the Universe work? (Quantum physics...)
  - What is Life? (e.g., quantitative systems biotechnology)
- Key Broader Challenges to Humanity:
  - Sustainable growth on earth. Sustainability means "change or die."
    - Global sustainable energy/environment & mid-term survival
    - "yin sustainability," e.g. population, related women's issues, peace
  - Cost-effective sustainable space settlement
  - Human potential -- growth/learning in brain, soul, integration (body)

Derivatives Made Real Physics Possible



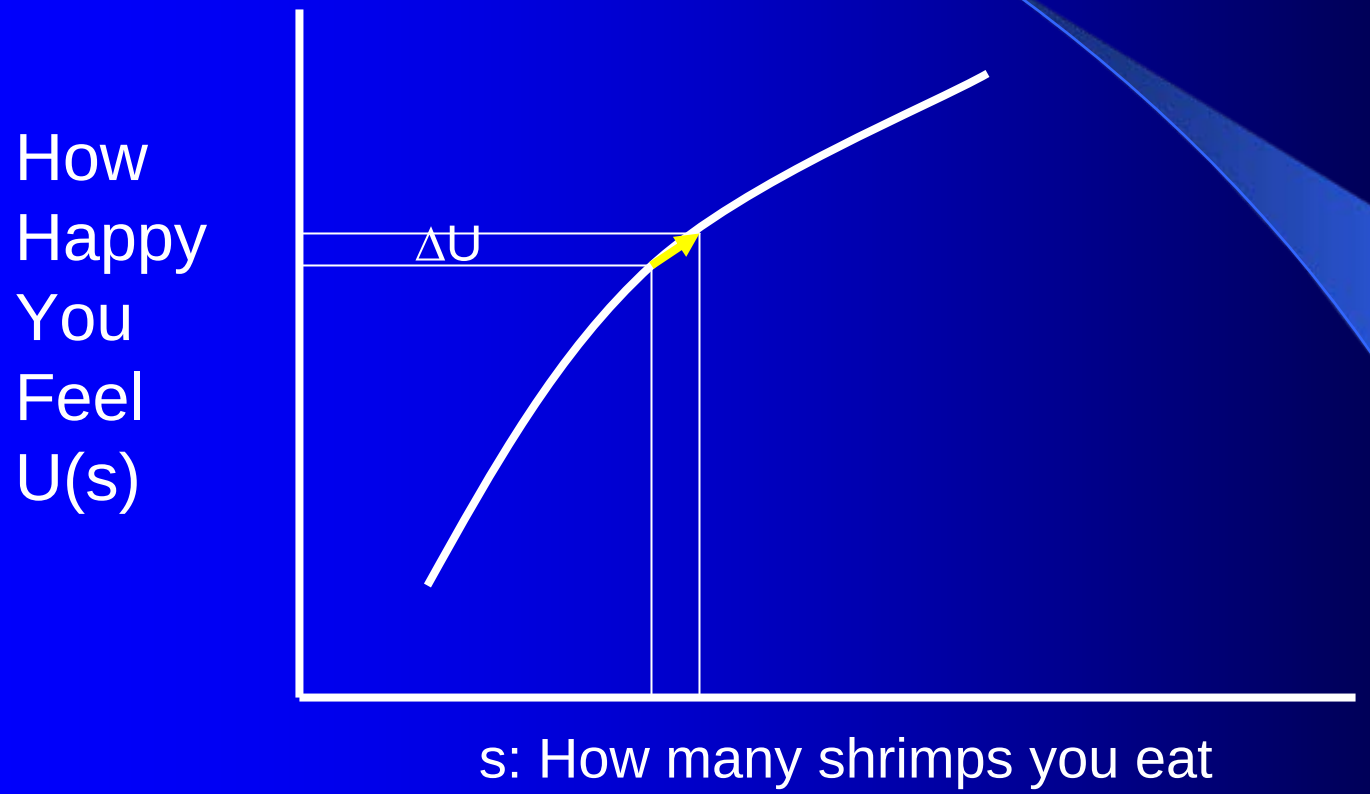
$$\frac{dh}{dt} = v \approx \frac{\Delta h}{\Delta t}$$

$$\frac{dv}{dt} = -32 \text{ feet per second per sec}$$

Height h is a function of time t, h(t)

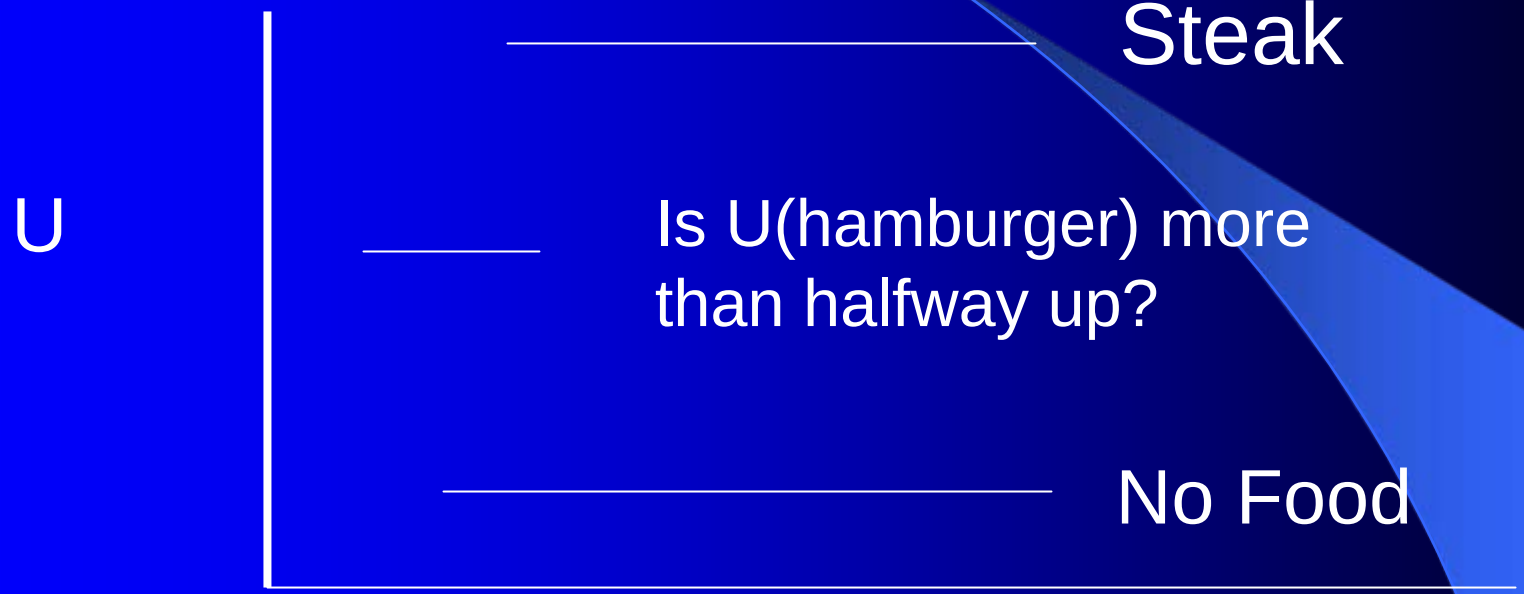
At the highest point, dh/dt = 0 – a flat arrow!

In perfect markets, prices are just derivatives of “utility” U



Value of a shrimp to you is ΔU, the extra happiness you get from eating it.
Economists call dU/ds the “marginal utility” of shrimp – just a derivative!
But your happiness is not just U(s); it’s a function of many variables…

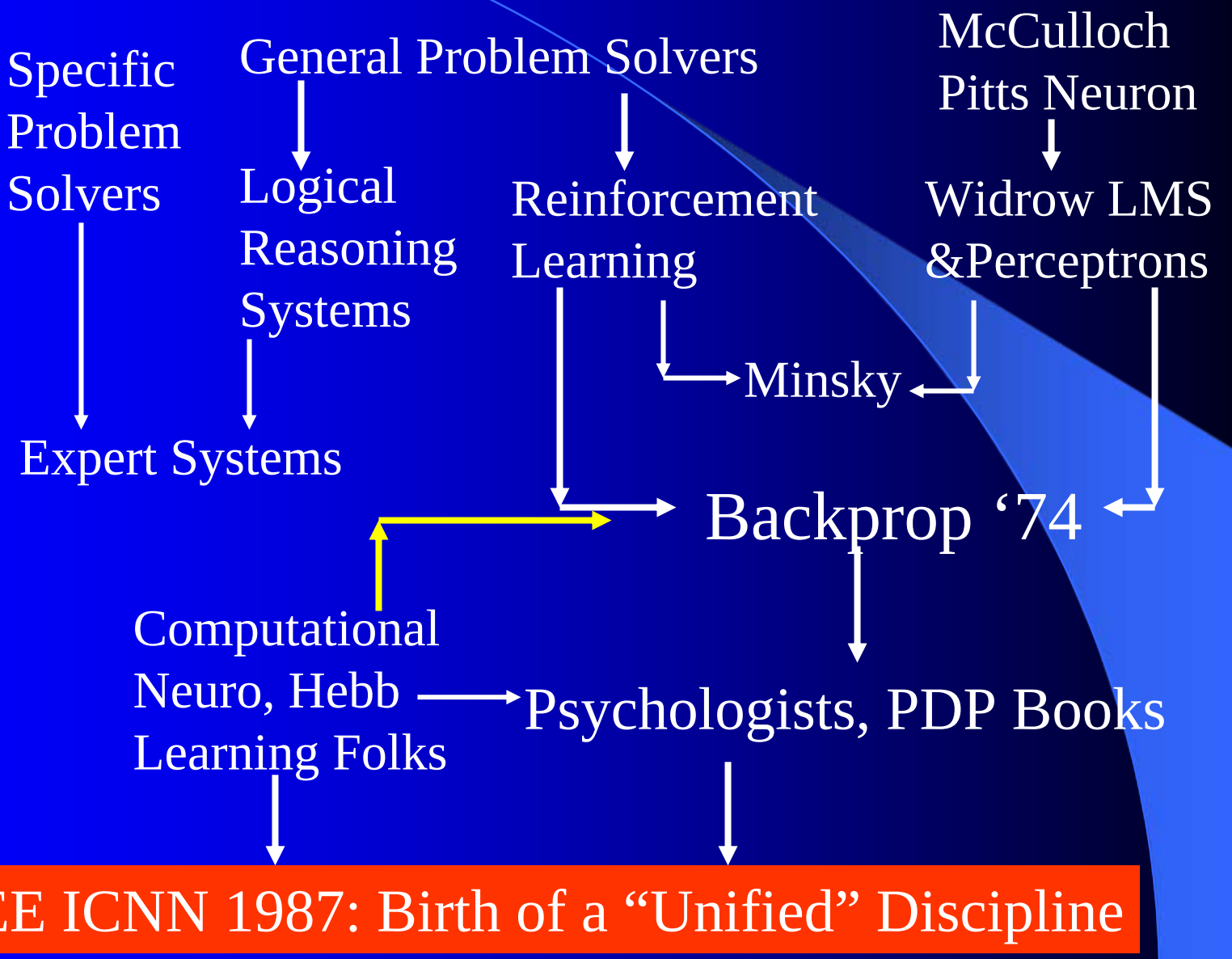
Von Neumann Showed that "Utility" is a Number, like Temperature



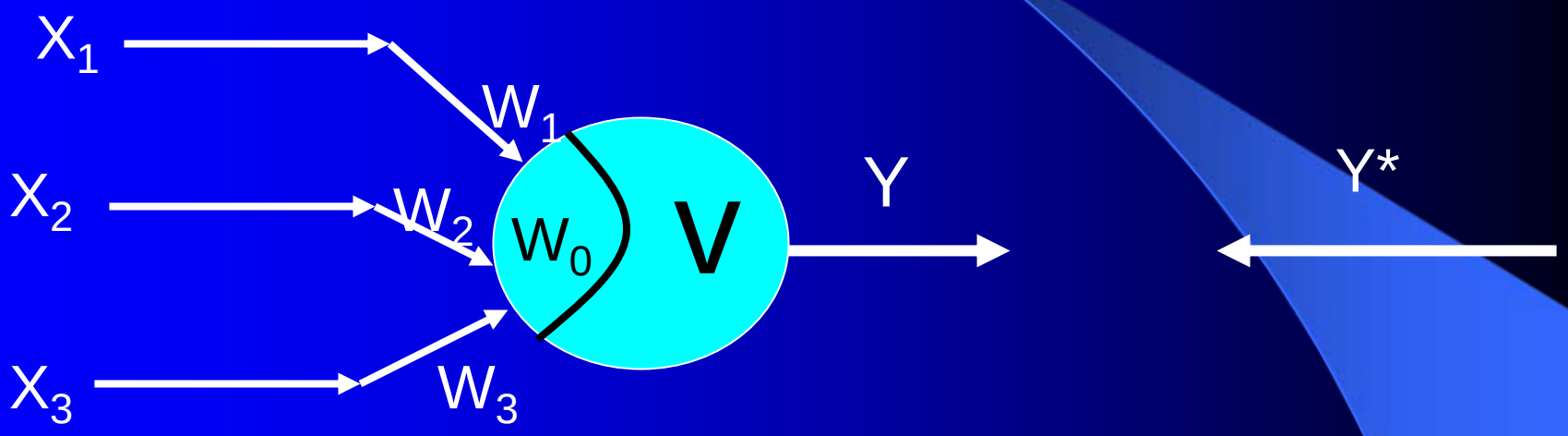
Von Neumann: just ask people to choice between certain hamburger, and 50-50 chance of steak versus nothing! (Modern decision analysis…risk management… etc.)

Can we understand how intelligence works in brains, well enough that we could write out equations, with equations good enough that we could put them on a computer and then the computer would be intelligent?

Where Did ANNs Come From?



Simplified Model of a Neuron



Voltage of cell: $v = W_0 + W_1X_1 + W_2X_2 + W_3X_3$

Output of cell (burst size): $Y = s(v)$

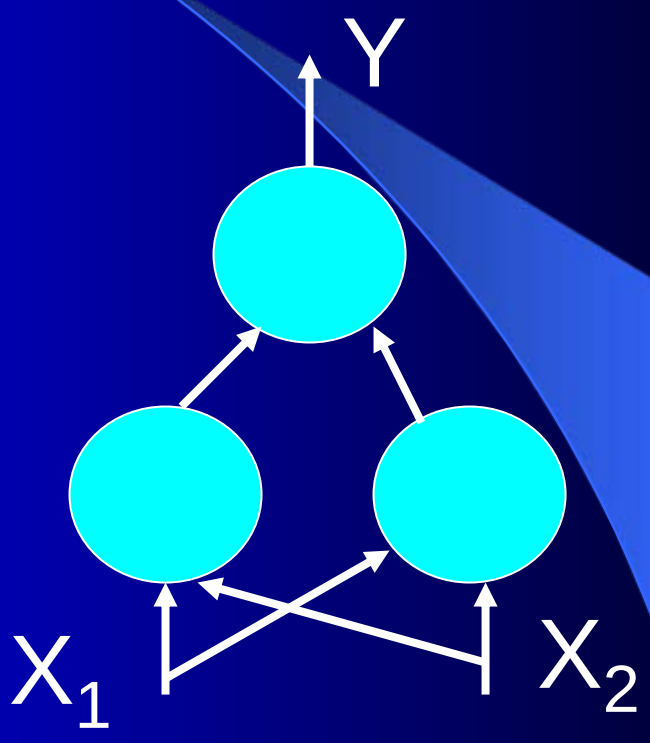
$$\frac{\partial v}{\partial W_i} = X_i \quad \text{for } i = 1,2,3$$

$$\frac{\partial Y}{\partial W_i} = \frac{\partial v}{\partial W_i} \cdot \frac{\partial s}{\partial v}$$

But some problems require more than one neuron….

The XOR function

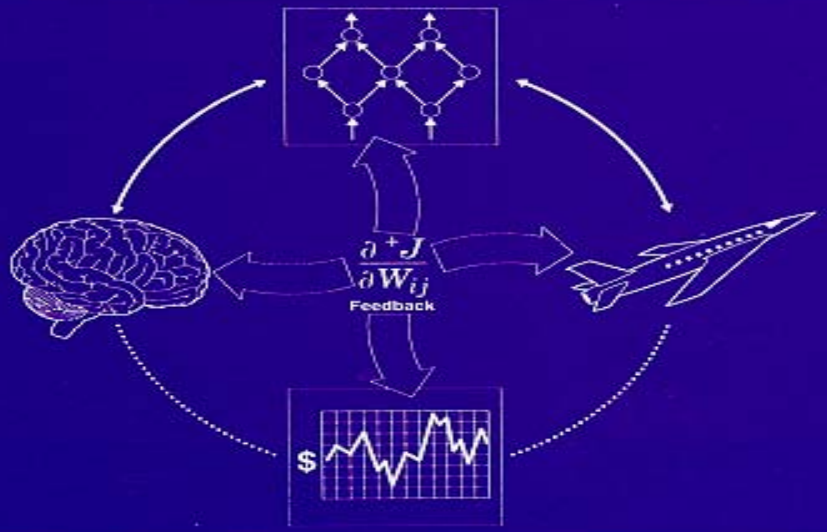
X_1	X_2	Y*
0	0	0
0	1	1
1	0	1
1	1	0



Minsky: How can we adjust the weights inside all three neurons, as needed to match the XOR function?

THE ROOTS OF BACKPROPAGATION

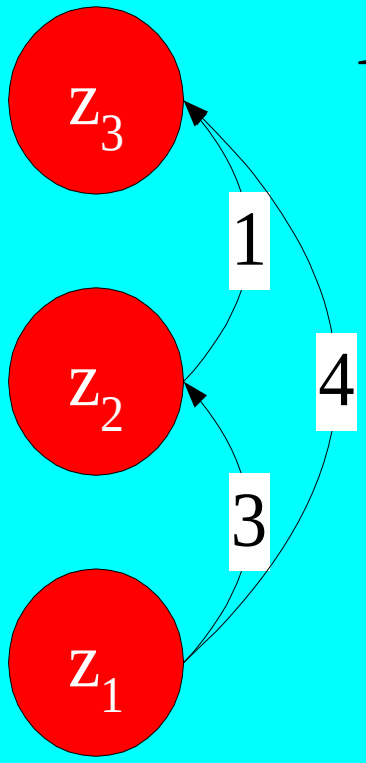
From Ordered Derivatives to Neural Networks and Political Forecasting



PAUL JOHN WERBOS

A Volume in the Wiley Series on ADAPTIVE AND LEARNING SYSTEMS FOR SIGNAL PROCESSING, COMMUNICATIONS, AND CONTROL
SIMON HAYKIN, SERIES EDITOR

How calculate the derivatives?



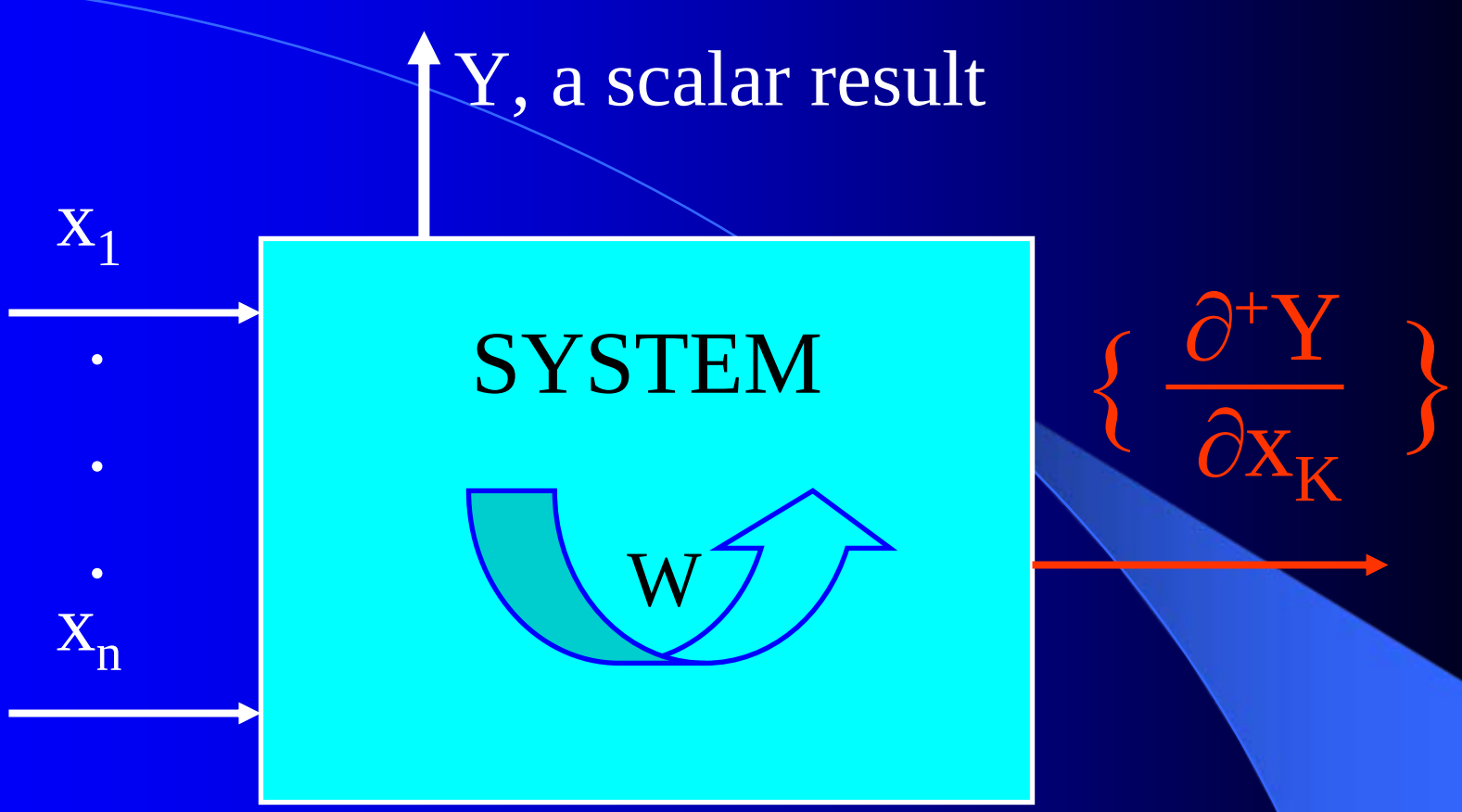
Dynamics $z_3 = z_2 + 4z_1; \quad z_2 = 3z_1$

$$\frac{\partial z_3}{\partial z_2} = 1 \qquad \frac{\partial z_3}{\partial z_1} = 4$$

$$\frac{\partial^+ z_3}{\partial z_1} = \frac{\partial z_3}{\partial z_1} + \frac{\partial^+ z_3}{\partial z_2}\frac{\partial z_2}{\partial z_1} = 7$$

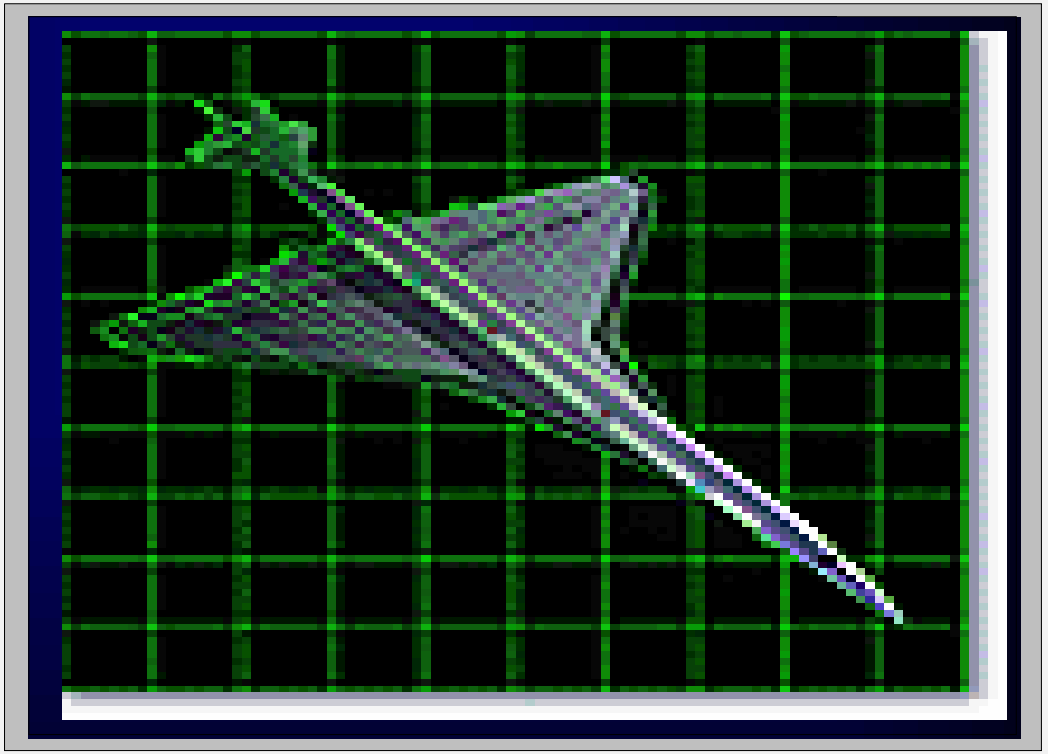
$$\frac{\partial^+ z_n}{\partial z_i} = \frac{\partial z_n}{\partial z_i} + \sum_{j=i+1}^{n-1} \frac{\partial^+ z_n}{\partial z_j}\frac{\partial z_j}{\partial z_i}$$

A Chain Rule For Ordered Derivatives

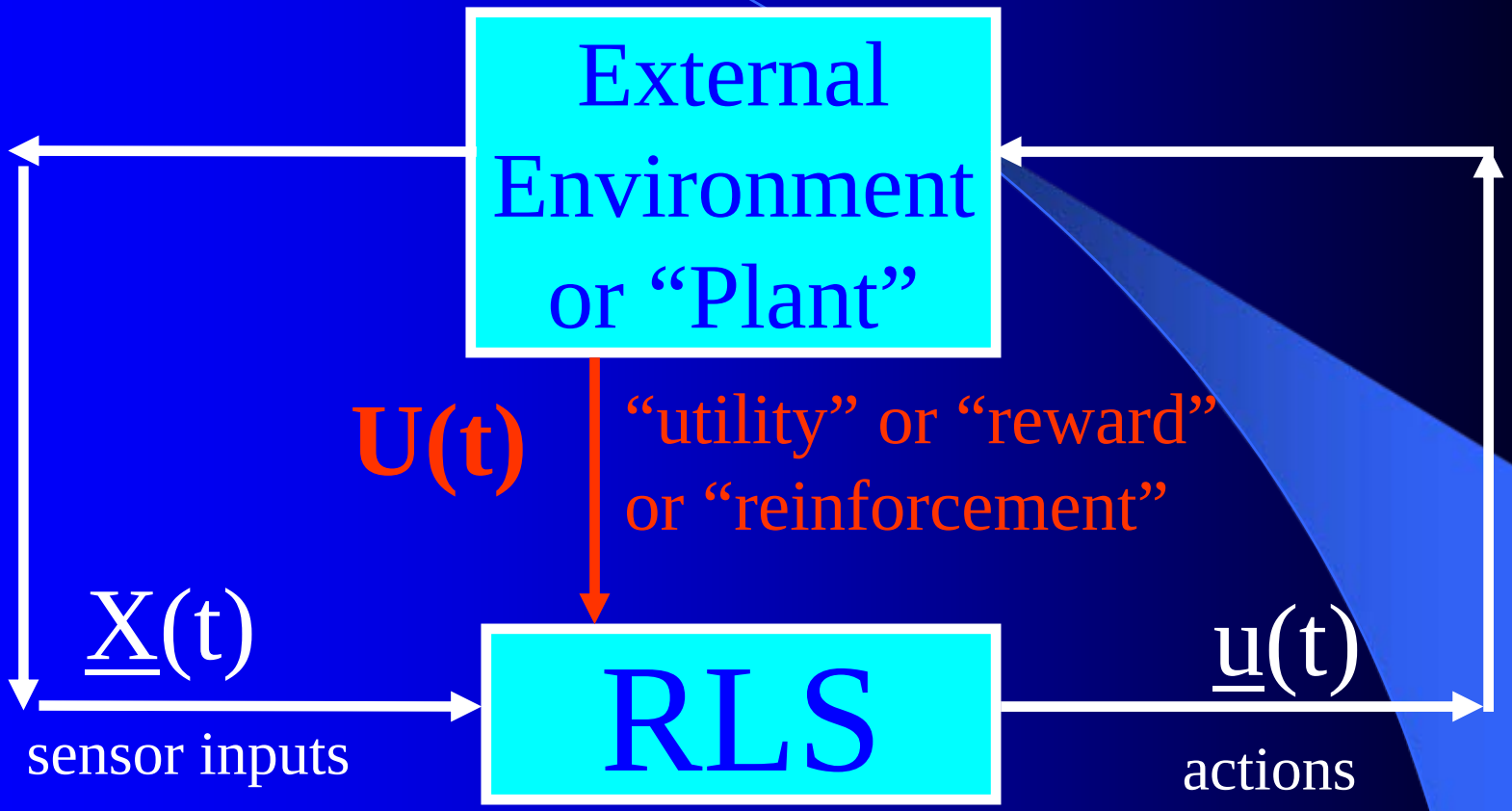


(Inputs x_k may actually come from many times)

Backwards Differentiation: But what kinds of SYSTEM can we handle? See details in AD2004 Proceedings, Springer, in press.



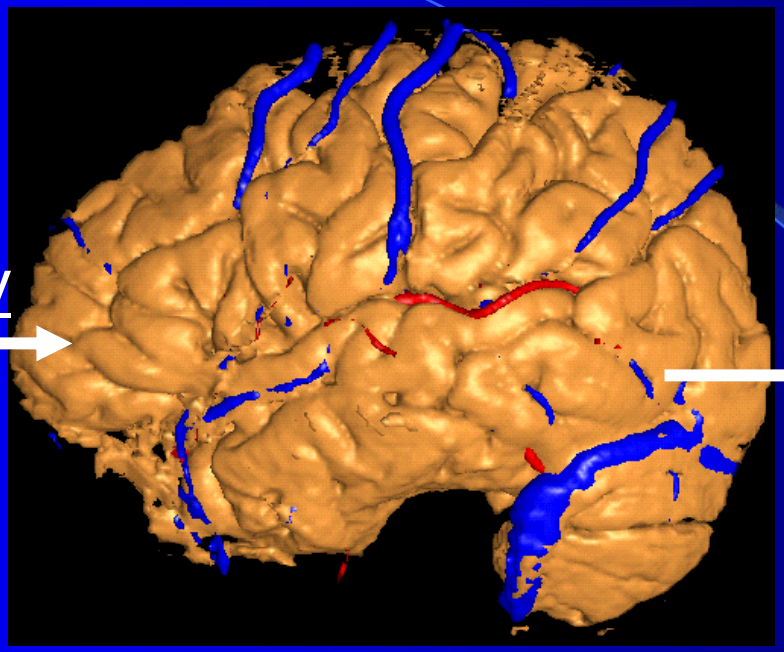
Reinforcement Learning Systems (RLS)



RLS may have internal dynamics and "memory" of earlier times t-1, etc.

Optimization & Prediction in Brain

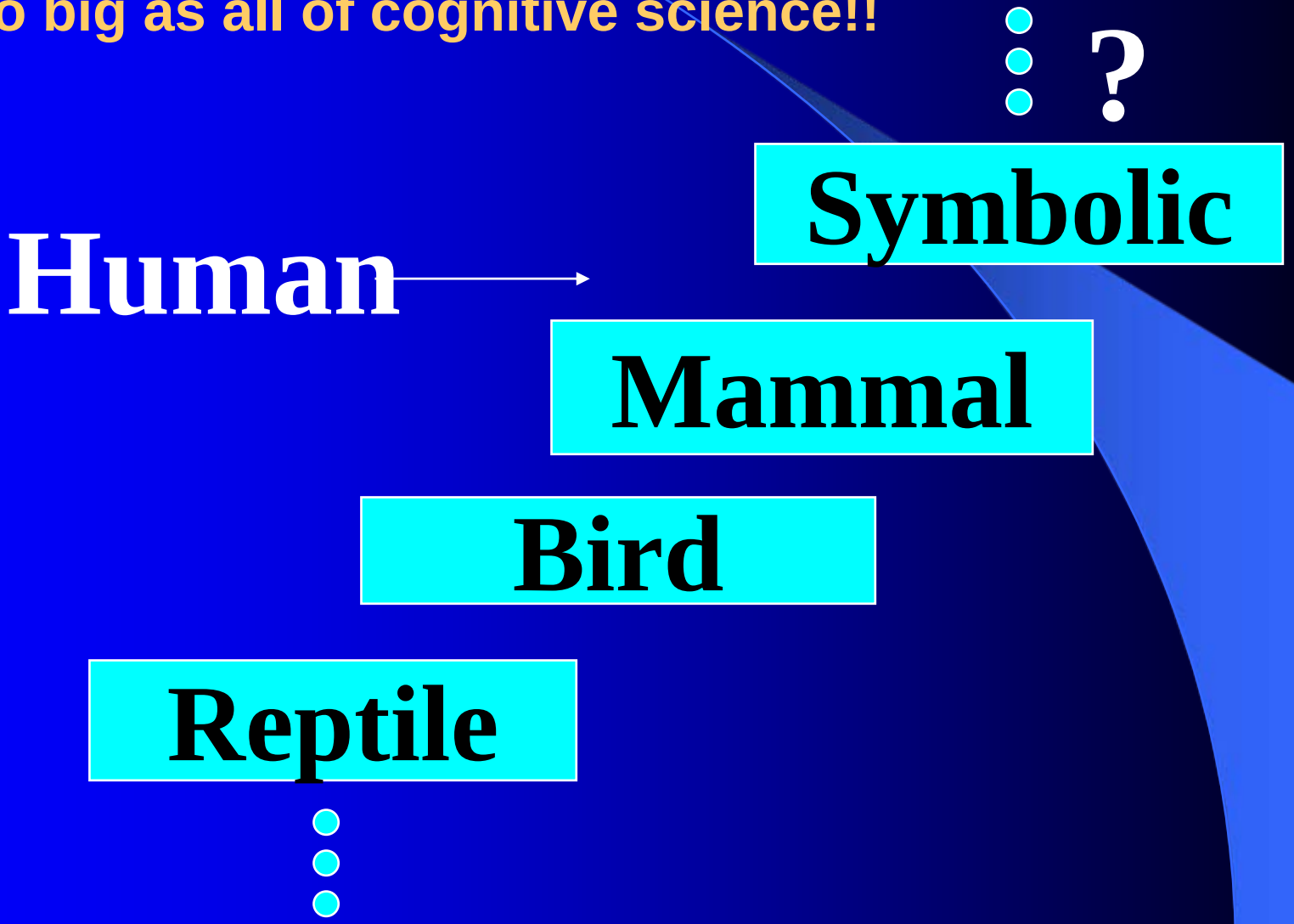
Sensor Inputs y



Motor Outputs u

From the fish to the smallest mouse, brains show amazing ability to learn how to maximize their long-term probability of survival under diverse, novel, complex circumstances, even before the evolution of unique human faculties like mirror cells, empathy, symbolic reasoning, etc. This is an optimization challenge. How do brains achieve and implement such a powerful and general optimization capability, using massively parallel hardware? Optimization also requires prediction. Goal: use engineers to reverse engineer prediction and optimization, from brain to usable design.

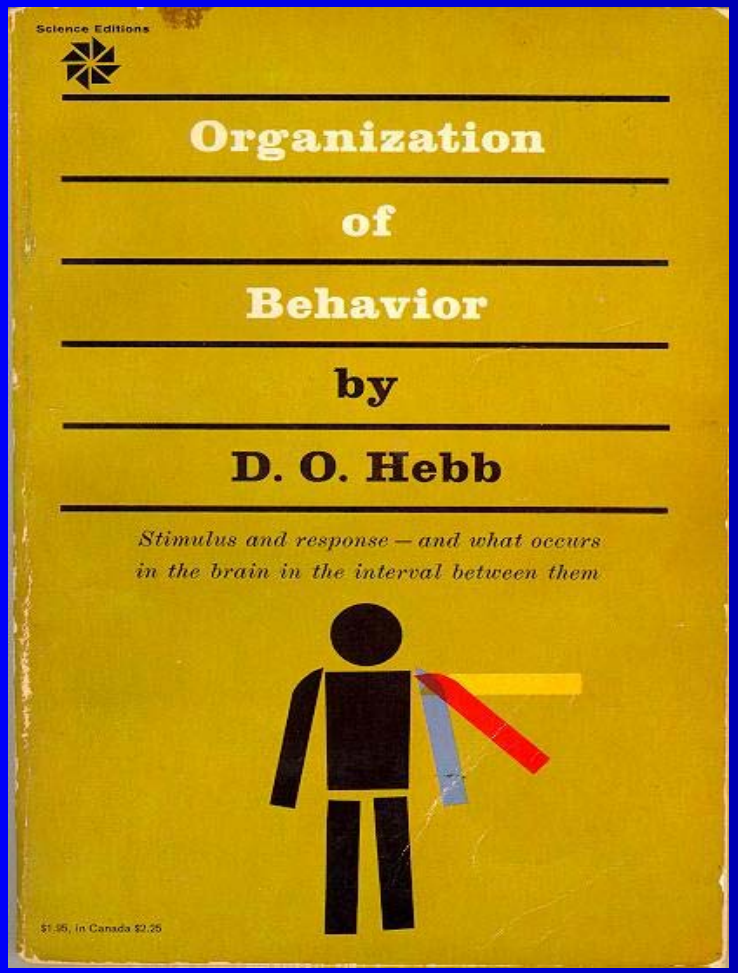
The Newtonian Revolution: mathematical understanding of **subsymbolic** intelligence, an essential step towards deeper understanding of the human mind and consciousness – but not so big as all of cognitive science!!



Optimality (III): Traditional Questions (&See papers at www.werbos.com)

- If brains are so optimal, why do humans do so many stupid things?
  - Brains are designed to learn to approximate optimal policy, as effectively as possible with bounded computational resources (neural networks), starting from a less optimal start. They never learn to play a perfect game of chess (nor will our computers) because of resource constraints. We just do the best we can.
  - When one human criticizes another, we are comparing two highly intelligent systems. Some brains learn faster than others, and humans are an intermediate stage towards even higher/faster intelligence.
- If the optimization theory is right, wouldn't brains get stuck in local minima?
  - They sure do. Everyone on earth is in a "local minimum," or a rut, to some degree. In other words, we could all do a bit better if we had more creativity. But look at those hairy guys (chimps) in the jungle, and the rut they are in!
  - The optimality theory says we combine an incremental learning ability with an ability to learn to be more "creative" – to do better and better "stochastic search" of the options available to us. (Widrow example.)

Hebb 1949: Intelligence As An Emergent Phenomenon or Learning

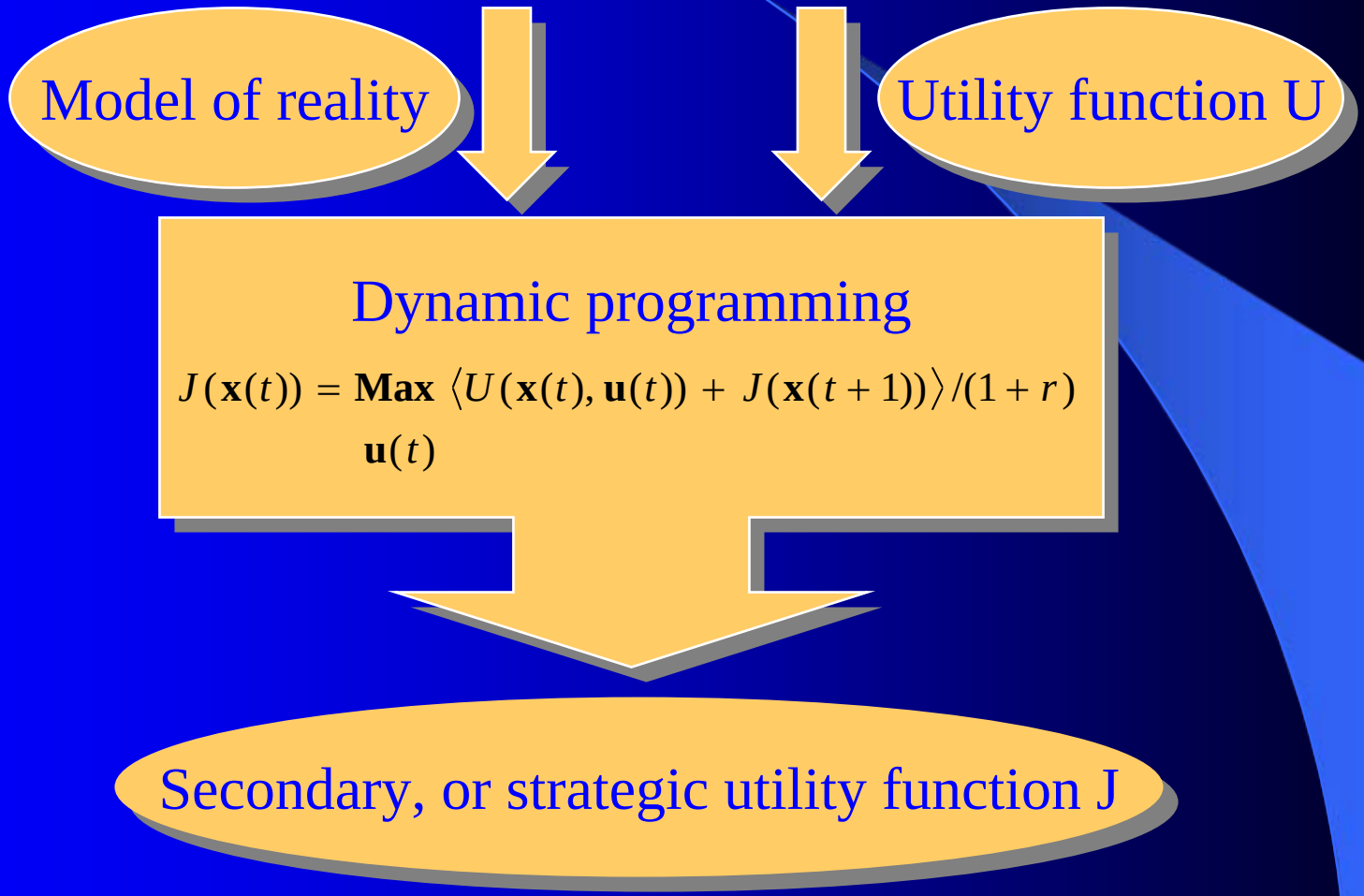


"The general idea is an old one, that any two cells or systems of cells that are especially active at the same time will tend to become 'associated,' so that activity in one facilitates activity in the other" -- p.70 (Wiley 1961 printing)

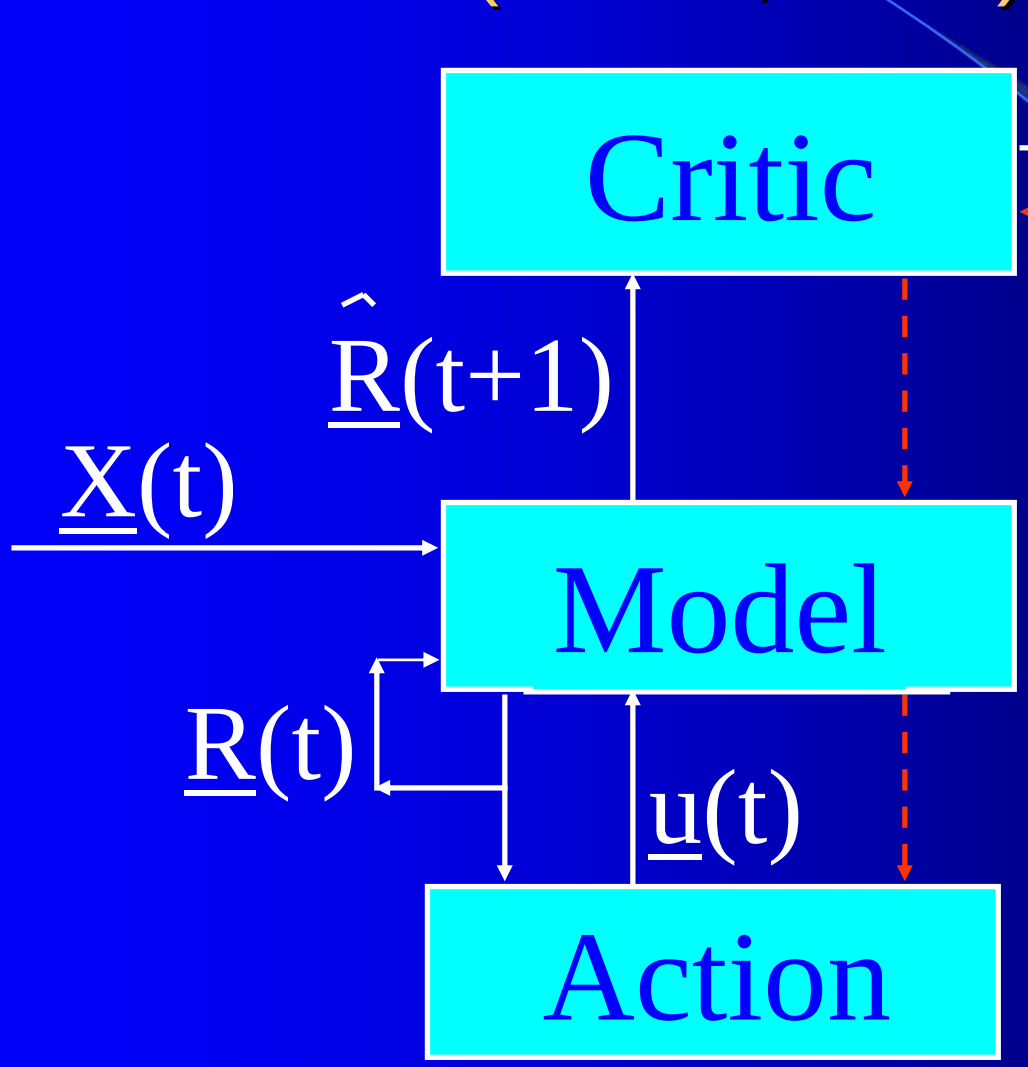
The search for the General Neuron Model (of Learning)

"Solves all problems"

Maximizing utility over time



1971-2: Emergent Intelligence Is Possible If We Allow Three Types of Neuron (Thesis,Roots)



Red Arrows: Derivatives Calculated By Generalized Backpropagation

Examples of J and U (and ∇U, ∇J)

DOMAIN	INTRINSIC UTILITY U	STRATEGIC UTILITY J
Chess	Win/Lose	Queen = 9 Points...
Business	Cash Flow, Profit	Net Present Value
Human Mind	Pleasure/Pain	Hope,Fear
Behavioral Psychology	Primary Reinforcement	Secondary Reinforcement
Artificial Intelligence	Utility Function	Position Evaluator
Economics (Derivatives)	Value of Product to You Now	Market Price or Shadow Price $\underline{\lambda}$

Venayagamoorthy/Wunsch/Harley ADP Turbogenerator Control

- Stabilized voltage & reactance under intense disturbance where neuroadaptive & usual methods failed
- Being implemented in full-scale experimental grid in South Africa
- Best paper award IJCNN99

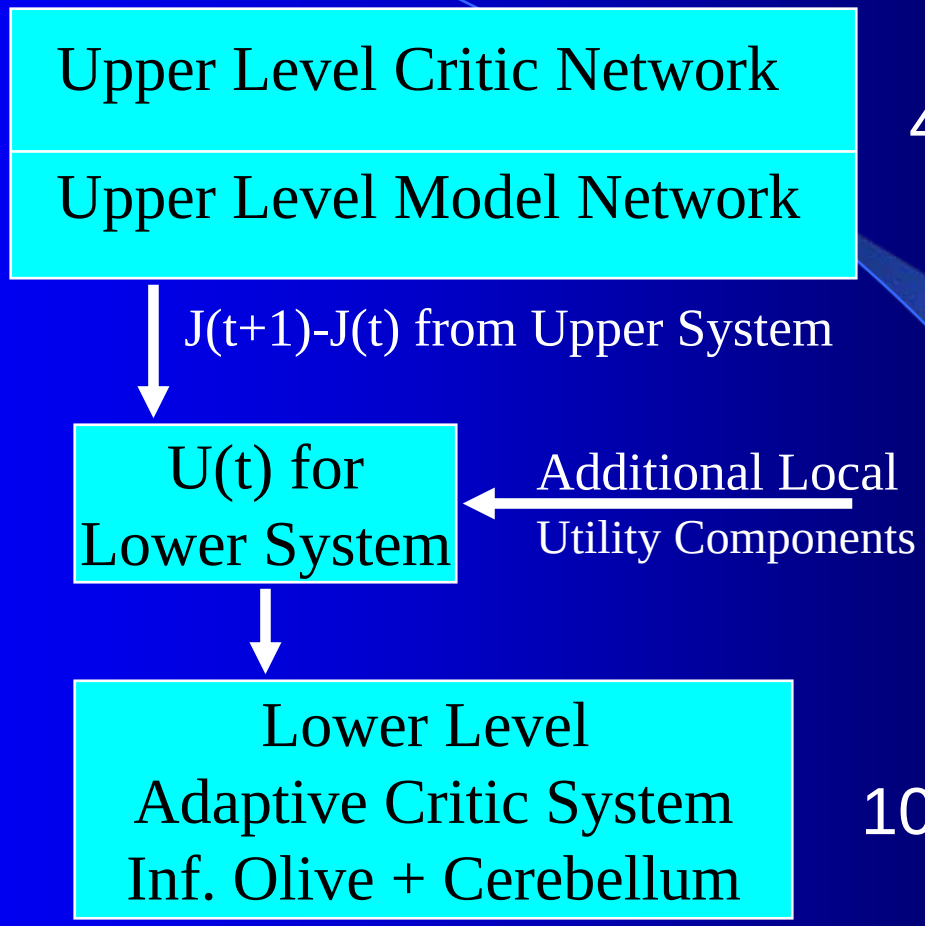
1st Generation Theory of Mammal Brain

- As in 71-72 proposal, brain has 3 main parts:
  - Cortex+thalamus: Model to predict/impute reality. See Nicolelis&Chapin, Science, rat whisker work.
  - Limbic system: Critic gives "emotional" assessment of what Freud called "objects" (Papez, James Olds)
  - Brain-stem: action or "motor" system (and inherited fixed preprocessors/postprocessors)
  - Clock signals from extracortical sources (Foote, Llinas)
  - Backprop unavoidable. (Bliss, Spruston, Sejnowski)
- Technical level improvements and big runs enough to span gap form 1971-72 to mammal brain:
  - Fill in "Model" with hybrid Simultaneous/Time-Lagged Recurrent Network trained by Error Critic (fully specified in Handbook of Intelligent Control)
  - Critic is sum of multiple "HDP" components each trained by GDHP, which gives power of DHP for continuous variables but handles continuous/discrete mix.
  - In each box, faster learning, per robust statistics, learning from memory, etc.
- BUT IS IT ENOUGH? For what?

Beyond Bellman: Learning & Approximation for Optimal Management of Larger Complex Systems www.eas.asu.edu/~nsfadp

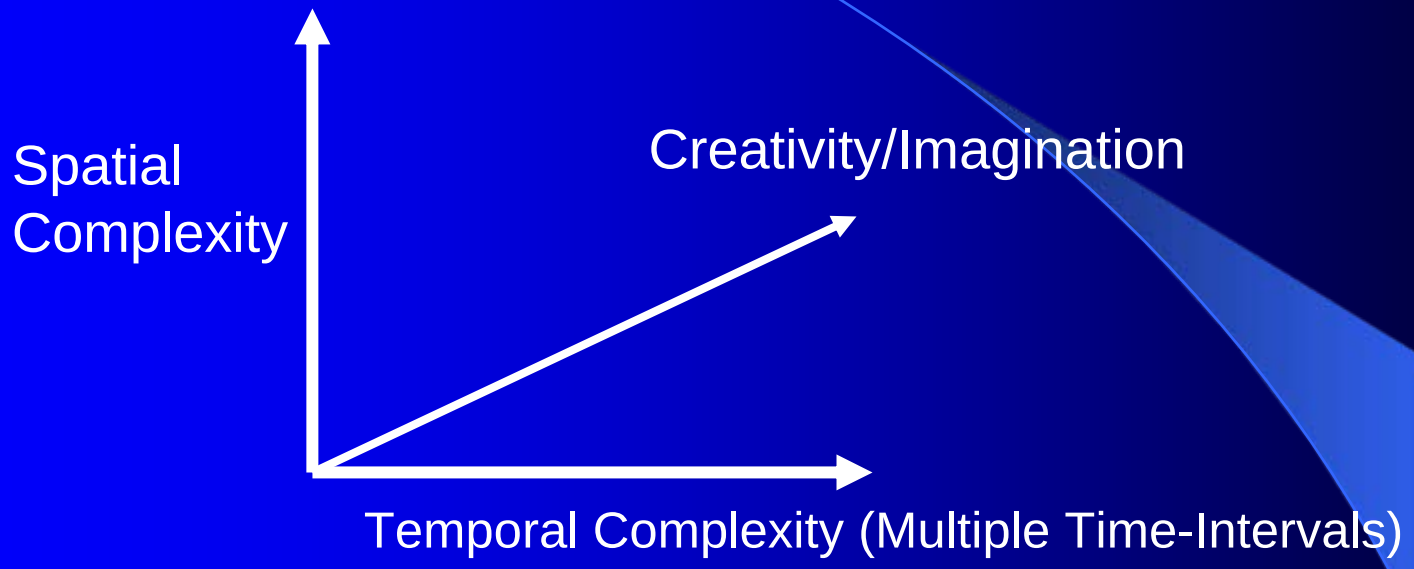
- Basic thrust is scientific. Bellman gives exact optima for 1 or 2 continuous state vars. New work allows 50-100 (thousands sometimes). Goal is to scale up in space and time -- the math we need to know to know how brains do it. And unify the recent progress.
- Low lying fruit -- missile interception, vehicle/engine control, strategic games
- Workshops: ADP02 & Dynamic Stochastic Grid testbed; ADP06 April 2006

2nd Generation "Two Brains in One Model"



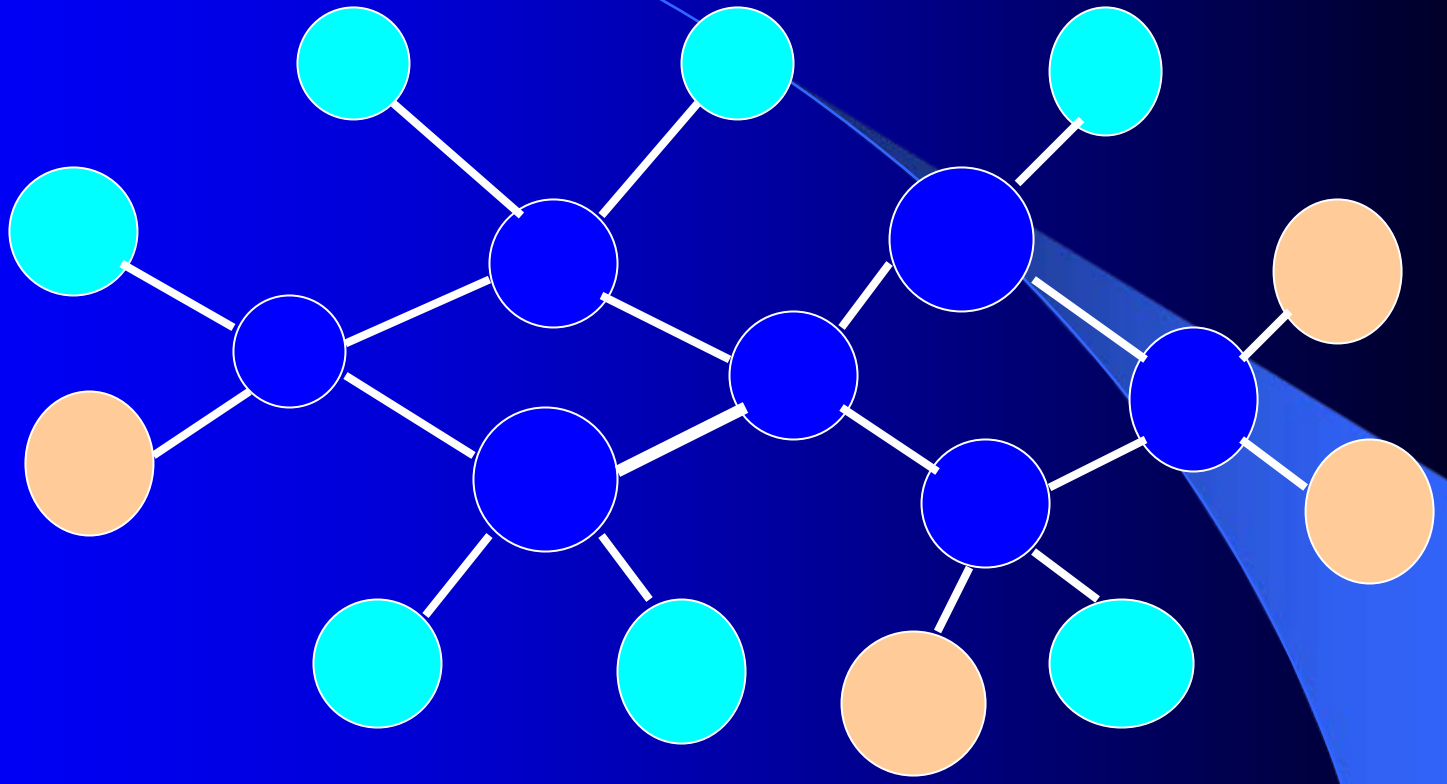
Concept in "Statistical/Numerical…", Trans. SMC, 1987 (on web)
Joint papers with Pellionisz (experimental follow-on still warranted)
See equations in Handbook of Intelligent Control, Ch. 13 & Prokhorov

Key Issues in 3rd Generation Model



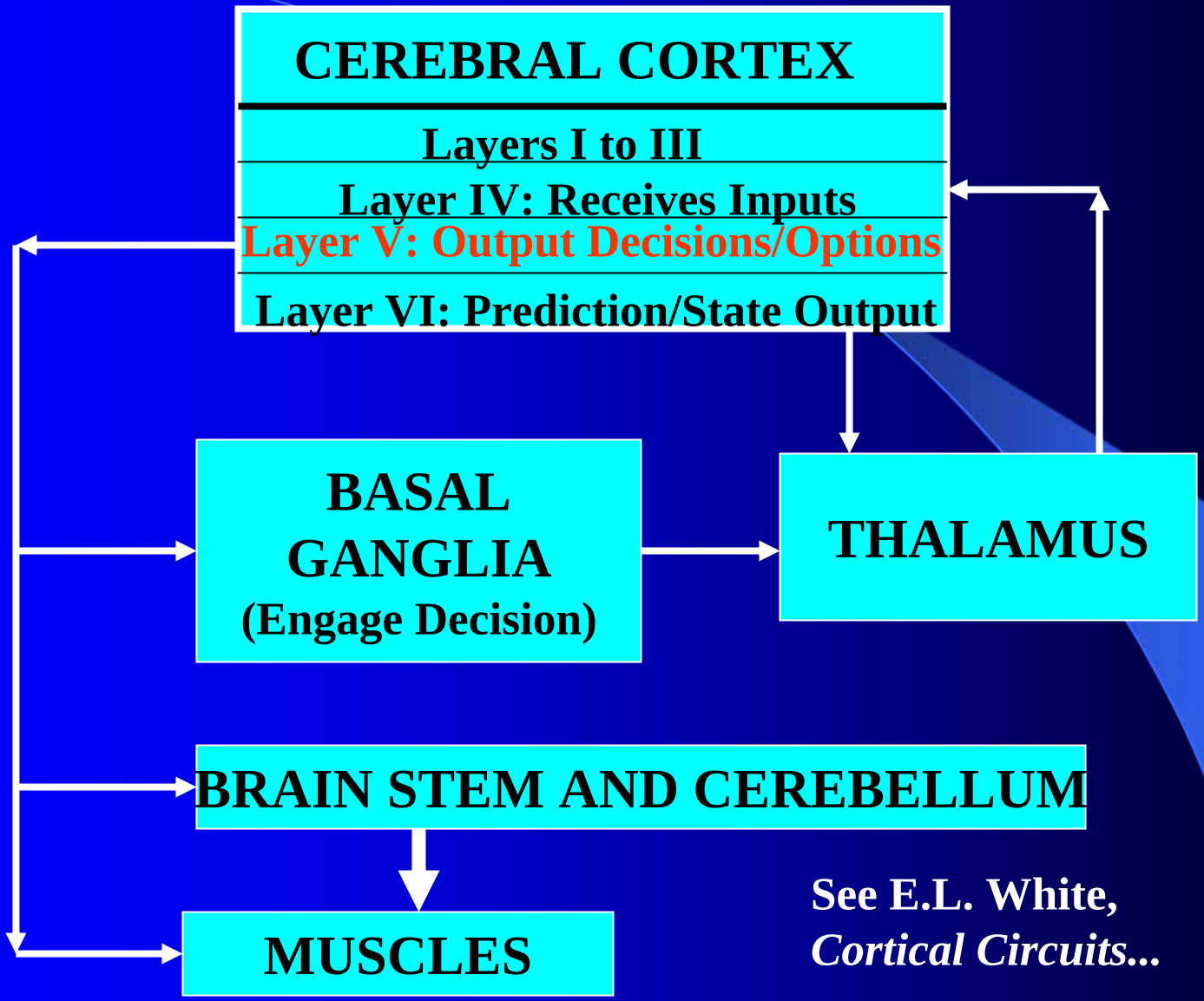
- Can we (and do brains) do better than 2nd gen brain in handling greater spatial & temporal complexity, by new designs & exploiting unspecialized but structured prior information (Kant) to get faster/better learning?
- What is our answer to AI's "spatial/temporal chunking" & stochastic search?
- All 3 demand more attention and work!!!

New Ways to Address Spatial Complexity Have Begun to Emerge…



- 4 General Object Types (busbar, wire, G, L)
- Net should allow arbitrary number of the 4 objects
- How design any universal nonlinear approximator to input and output FIELDS -- variables like the SET of values for current ACROSS all objects?
- Great preliminary success (Fogel's Master Class Chess player; U. Mo. Power). Go next?
- But how learn the objects and the symmetry transformations???? (Brain and images!!)

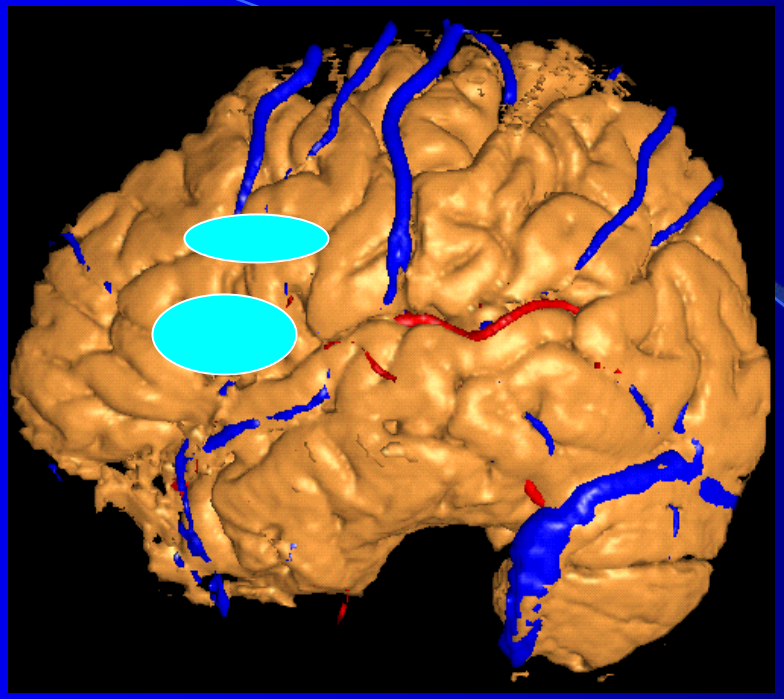
3rd Generation View of Creativity/Imagination: Layer V = "Option Networks"



See E.L. White, *Cortical Circuits...*

- Challenge: www.werbos.com/WerbosCEC99.htm.
- Important work by Serpen, Pelikan, Wunsch, Thaler, Fu – but still wide open. Widrow testbed.

New Data on Complexity in the Brain



Petrides (IJCNN06) shows that dorsolateral (DL) and orbitofrontal (OF) prefrontal cortex – our “highest” brain centers – answer two basic questions:
OF: Where did I leave my car this time in the parking lot? (space?)
DL: What was I trying to do anyway? (time?)

- BUT: even bird brains (no neocortex) handle great spatial complexity & have big basal ganglia!!
- Hypothesis: SEDP fits pyramid cell geometry very well but is already be in old cortex (bird!)
- Neocortex (mouse) harnesses/alters stochastic mechanism in SEDP for creativity.
- OF strengthens object identity & world modeling & object-oriented action. (Test birds, lizards!)
- Temporal aggregation is by “re-entrant” mechanism, not explicit temporal hierarchy.