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Analog quantum computing (AQC) and the need for time-symmetric physics

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Abstract This paper discusses what will be necessary to achieve the full potential capabilities of analog quantum computing (AQC), which is defined here as the enrichment of continuous-variable computing to include stochastic, nonunitary circuit elements such as dissipative spin gates and address the wider range of tasks emerging from new trends in engineering, such as approximation of stochastic maps, ghost imaging and new forms of neural networks and intelligent control. This paper focuses especially on what is needed in terms of new experiments to validate remarkable new results in the modeling of triple entanglement, and in creating a pathway which links fundamental theoretical work with hard core experimental work, on a pathway to AQC similar to the pathway to digital quantum computing already blazed by Zeilinger's group. It discusses the most recent experiments and reviews two families of alternative models based on the traditional eigenvector projection model of polarizers and on a new family of local realistic models based on Markov Random Fields across space–time adhering to the rules of time-symmetric physics. For both families, it reviews lumped parameter versions, continuous time extension and possibilities for extension to continuous space and time.

Keywords Analog quantum computing · Triphoton · Quantum measurement · Stochastic quantization · Markov Random Fields · Spintronics · Polarizers · Bell's Theorem experiments

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1 Introduction

For purposes of this paper, analog quantum computing is defined as the complete spectrum of activities, from computational theory to physical building of hardware and experiments, needed to enable the richest design space for computational systems, including hybrid systems intended for intelligent control, imaging and communications.

More precisely, a computational hardware design can be represented by a graph of available hardware components implementing mathematical functions. Traditional classical computing focused on the design space of functions inputting binary strings and outputting binary strings, and analyzing what functions from string to string can actually be implemented. Even within that portion of the larger design space, David Deutsch proved [1] that systems exploiting certain basic unitary quantum operations can implement functions which are a superset of those available to classical computers, short of an exponential cost in simulating quantum systems on a classical computer. But modern engineering and industry address an even larger set of functions, entailing a mixture of continuous and discrete variables, and also requiring attention to issues of function approximation [2] and to the implementation of stochastic maps, not just deterministic ones.

Continuous-variable quantum computing [3,4] already offers a very important generalization of the original Deutsch version of quantum computing, a superset of Deutsch's superset [1], if both continuous and binary variables are handled in hybrid mode. The goal of AQC, as defined in this paper, is to extend that work, by also adding the ways of function approximation based on the most advanced work done the neural network and function approximation communities [2,5], as well as adding capabilities to model the behavior of circuits and incorporate the stochastic effects. Many are now aware of the stochastic search capabilities of new systems ranging from Hopfield networks implementations [6] to traditional evolutionary computing to quantum simulated annealing used in D-Wave [7]. However, stochastic elements are also essential to the most powerful new tools for intelligent control as described in the opening chapter of [8] and in [9], and for understanding of intelligence in the brain [10,11].

This paper will focus on the new experiments and basic understanding necessary to perform useful hardware design of this kind of stochastic quantum computing, within the realm of continuous quantum computing. This is not intended as an alternative to the many useful things which can be achieved with deterministic, unitary mappings in continuous-variable quantum computing; rather, it is intended to enrich continuous quantum computing by adding our ability to incorporate the stochastic elements into the quantum computing circuit and model them. More precisely, the goal is to consider the role both of stochastic and of dissipative elements within a quantum circuit. To get started in this direction, the most essential circuit elements that we need to model properly at a macroscopic level are the continuous or tunable spin gates. Spintronics using spin gates is already seen by the electronics industry as a major practical step forward, while unitary or adiabatic computing [12] was around for a while still retaining its relevance, and both areas of research are important parts of the long-term roadmap. It is crucial to remember that classical gates in electronics (transistors) and the practical spin gates worked on today are dissipative systems. Tunable spin gates are essentially just extensions of the tunable linear polarizer, a basic circuit element which has been central to the progression in the laboratory of Zeilinger [13], progressing from empirical work on Bell's Theorem experiments, up to empirical work on discrete triphoton experiments, producing Greenberger, Horne and Zeilinger (GHZ) states and from there to empirically well-grounded work on quantum computing hardware. Here, we try to set the stage for a similar progression, within the realm of stochastic continuous-variable quantum computing.

There are three areas of technology which would need to be developed to get the broadest possible use of AQC in computing and in communication: (1) systems level information technology, algorithms and architectures; (2) methodology of dealing with decoherence, disentanglement and dissipation; and (3) the basic device design, including modeling, necessary for building large quantum systems. This paper will focus mostly on (3), since we have covered the first issue extensively elsewhere [14,15]; and the second issue is so important by itself that it deserves a separate discussion.

Finally, for this special issue, I would like to express my great thanks to Howard Brandt who, together with Marlan Scully and Yanhua Shih, played an essential role in the work leading up to this paper. From reviews of quantum computing at National Science Foundation (NSF), I learned long ago that quantum computing systems can only work in the real world of condensed matter physics when they are designed based on a density matrix formulation of how our knowledge actually evolves in time in quantum mechanics [16]. At the interagency QISCOG meetings on quantum computing, Howard Brandt explained that to me, encouraged my early thoughts on how to build quantum learning machines (which I funded for several years as a Program Director of NSF), and pointed me toward two books which played an essential role in my understanding of those issues [17,18], going well beyond what I had learned in graduate school from courses by Schwinger and Coleman, valuable as those also were. Howard specifically reviewed, encouraged and accepted my first paper on the proposed new continuous triphoton experiment [19] for the SPIE conference on quantum computing in 2015. It was a great shock to arrive at that conference and see, not Howard, but memorial talks for him.

2 Analog quantum computing: development of options for the physical platform

2.1 The need for dissipative circuit elements

Spin gates are inherently dissipative objects, because energy is lost when a photon is absorbed. For optical dissipative systems, it is well known that density matrices are essential to describing our knowledge about a quantum system [16–18,20]. In fact, even standard digital quantum computing became realizable in hardware only after the original theoretical concepts involving wave functions were mapped into the practical world of density matrices to describe physical hardware [21].

An ideal linear polarizer is dissipative, but not stochastic, if all photons entering it are either aligned to its preferred orientation, or orthogonal to it. However, as soon as we build systems, which include photons of arbitrary orientation, it becomes stochastic. For example, if a photon enters a polarizer at a 45° angle, there is a 50 % probability that it will be absorbed, and a 50 % probability that it emerges with a linear polarization equal to the preferred orientation of the polarizer. By contrast, ideal beam splitters like ideal wires are not dissipative and not stochastic, though they are important support elements from AQC in general. In the standard density matrix formalism, that it represented by saying that the input photon is described by a wave function representing a definite 45° polarization, while the output result is the weighted sum of two density matrices, one representing a state of zero occupancy of the number of photons and one representing a single photon polarized along the preferred orientation. It is crucial to remember that all of the successful predictions of actual Bell's Theorem experiments used this standard projection theory of what a linear polarizer does. More concretely, the quantum mechanical predictions shown in Eq. 4 were derived in [19] by assuming that each polarizer projects the eigenfunction of the system into an eigenfunction of the polarizer for the photon passing through the polarizer, exactly as Scully does in successful concise derivation of the usual quantum mechanical predictions for Bell's Theorem experiments [20].

2.2 From photons to other photon-like spins

In order to make AQC real, we need a strategy for how to get started in the physical implementation. As with digital quantum computing, there are many physical substrates that could be used, but quantum optics is one of the most promising [22], especially if our goal for now is to build things, which work as soon as possible. Yanhua Shih has previously shown [23] how to entangle real numbers, such as momentum, but computing with entangled spins currently seems a lot easier to work with.

In the long term, there are many particles, beyond photons, which may provide more computational throughput for their cost, once the basic principles are proven out and we are ready to aim for smaller feature sizes and massive scale. For example, there are many concepts of "spintronics" of great interest to the electronics industry [24]. Some of these concepts involve computing with the spins of plasmons or polaritons, which allow much smaller feature sizes than traditional photonics, but involve the same general principles. For a rational research program, it would make sense to include some funding today of efforts to perform basic experiments with entangled polaritons, similar to what has already been done long ago in macroscopic quantum optics. For example, it would be exciting to see Bell's Theorem experiments or GHZ experiments for entangled polaritons, or other spins, in a totally solid state system. But if we want to build interesting systems as soon as possible, our best hope lies in building upon the more complete entanglement using photon spins, for which the states of quantum entanglement were already produced in quantum optics.

For analog spintronics in general, it is clear that people will need to build up circuits with many core components, the most important of which would be spin gates, playing a role analogous to that of a transistor in classical computing. The Joint Quantum Institute (at University of Maryland, College Park) has already developed a digital switch for optical qubits [25], meeting the needs for first generation quantum computing. But for AQC, we need to have some kind of continuous tuning capability. We need to have gates, which can be tuned to a continuum of possible angles, as in polarizers or rotators [26,27]. Even though we do not yet have a consensus on exactly what circuit elements we will need, we do know we should at least be able to predict the behavior of simple circuits, which contain a handful of continuously tunable polarizers. That is not a sufficient condition of AQC, but it is a crucial prerequisite. We cannot effectively design even simple circuits unless we can correctly predict how they will behave. That is a crucial gap in our present knowledge, which we are now ready to fill in.

Let me make an analogy here to explain this better. The three groups which are capable of entangling three or more photons in GHZ states [13] are Zeilinger's group in Austria, Yanhua Shih's group at the University of Maryland, Baltimore County, and the new group led by one of Zeilinger's former students in Sichuan province, China. All three groups used Spontaneous Parametric Down Conversion (SPDC) to produce entangled photons, a technique first pioneered by Shih for use in Bell's Theorem experiments. Zeilinger's publications show a massive, systematic spectrum of work, from early Bell Theorem experiments up to more and more entangled photons, moving in a smooth way toward ever more serious quantum computing with qubits. It merges theory and experiment in a way, which we all know is desirable, but not always so easy to do. The challenge before us now is to start building the same kind of pathway for the more general case of AQC, where spins are treated as continuous variables and where polarizers (and other components) are continuously tunable. This will require a whole spectrum of new experiments, building from the initial work already started [28,29], coupled to the new theoretical work which will be reviewed in the next section.

3 Challenges in predicting analog networks of entangled photons and polarizers

3.1 Bell's Theorem and continuous triphoton examples

Building up on my paper [19] at the conference on Quantum Information and Computation at SPIE in Baltimore, 2014, organized by Howard Brandt, I did quite a bit of further work on the prediction challenge for triple coincidence detection rate for triphoton experiments for two different models. This work is already available in journals and Conference proceedings online [30–32] and in [33]; however, this material is a complex network in itself, so it makes sense to give an overview here, to explain how the pieces fit together.

The simplest design of the network of entangled photons and tunable polarizers, representing the classic Bell's Theorem experiments, is illustrated in Fig. 1, where the polarizer on the left is tuned to a linear polarization of θ_a, the polarizer on the right to θ_b, where the γ variables are logical variables (true or false) indicating presence or absence of a photon on the left channel (L) or right channel (R) either before (−) or after (+) the polarizer, and where the θ variables over the arrows represent the linear polarization of the photon on corresponding channel if the photon is present.

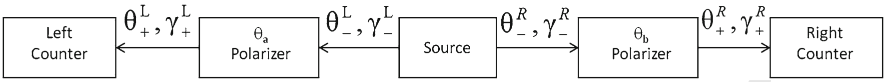

Fig. 1 Core structure and notation for the Bell's Theorem experiments

The paper [19] presents two alternative types of model, which provide the correct prediction for these experiments. More precisely, the two competing models both give the correct prediction for the curve $R_2/R_0(\theta_\mathrm{a}, \theta_\mathrm{b})$, the ratio for the two-photon coincidence counting rate R_2 over the two-photon production rate R_0 as a function of the two angles θ_a and θ_b over the full continuous range of angles. Both of the models can be described as "lumped parameter" models, in which the entire history of events is represented through a finite set of discrete and continuous variables.

The first model was the usual standard prediction of quantum mechanics, taken from the historic literature on Bell's Theorem experiments. Scully and Zubairy [20] review the same basic concepts in section 18.3 of [20], in a form concise enough that the calculations are left as an exercise for the student. A key part of those predictions is the assumption that the polarizer constitutes a dissipative element of the circuit, unlike mirrors and beam splitters, so when the light is propagating through dissipative media the projection of the wave function occurs, which if having been observed would have represented a collapse of the wave function.

Another model presented there was of a totally different kind, a discrete Markov Random Field (MRF) Model, which treats the network represented in Fig. 1 as a classical Bayesian type network of probabilities. Let us define a scenario or path X as a set of values for the eight variables you can see in Fig. 1. We get the correct predictions for R_2/R_0, the ratio for the two-photon coincidence counting rate R_2 over the two-photon production rate R_0, if we assume that

$$P^*(X) = p_1(X)\, p_2(X)\, p_3(X) \ldots p_n(X) \tag{1}$$

$$\Pr(X) = P^*(X)/Z \tag{2}$$

where the partition function Z is a normalization constant, and the relative or nodal probabilities $p_\mathrm{k}(X)$ are based on models for the type of object which appears in node k of the graph of the experiment. Here, there are five nodes, $n = 5$, and three types of node. See [19] for the specific models of each type of node, and the constraints that led to a workable model.

It is not necessary to overly rely on black box models of an object, such as polarizer, for either type of model, traditional or MRF. After all, we do know something about the physics of how polarizers do their job. Thus, a slightly more complicated model was developed, MRF3, of what actually happens in a Bell's Theorem experiment, in the case where calcite polarizers are used. In MRF3, the picture in Fig. 1 is made more complex, by replacing the right channel, for example, by the graph shown in Fig. 2, where the polarizer on the right is still tuned to a linear polarization of θ_b, where the γ variables are logical variables (true or false) indicating presence or absence of a photon on the right channel (R) either before (−) or after (+) the polarizer, and where the θ variables over the arrows represent the linear polarization of the photon on the channel

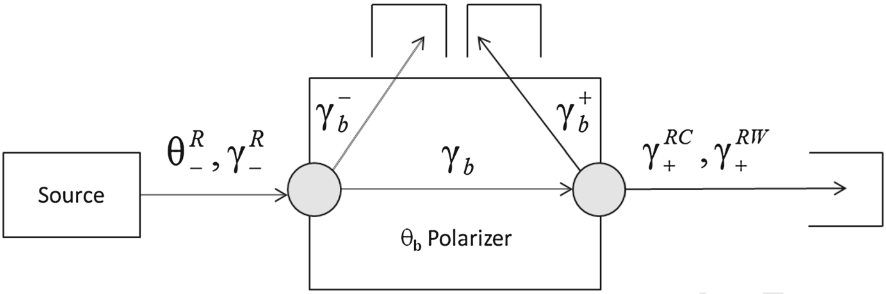

Fig. 2 Higher-resolution graph of the right channel of the experiment

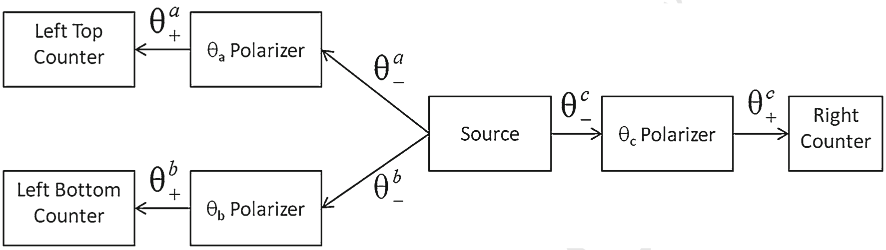

Fig. 3 The general analog triphoton experiment

before the polarizer if the photon is present. In this model, the outgoing photon has only two possible polarizations of significant probability, a clockwise circular polarization (RC) and a counterclockwise polarization (RW).

This model was much closer to the known physics than either of the two previous models, in my view. The most important question raised by this work was: what happens when we go just one step beyond, to try to predict the outcomes of the same experiment, modified by using three entangled photons? While others have previously proposed and performed experiments involving three photons [13,29], I believe that I was the first to propose and predict the general continuous triphoton experiment [19,32] illustrated in Fig. 3, where θ_a, θ_b and θ_c are the angles which the polarizers are tuned to (as in Fig. 1), where the θ variables over the arrows represent the polarization of the photons traveling on those channels, where the γ variables in Fig. 1 are still used in the calculations but not shown here just for simplicity, and where the source emits the GHZ state defined by

$$\psi = \frac{1}{\sqrt{2}}\left[|0\rangle_a |0\rangle_b \left|\frac{\pi}{2}\right\rangle_c + \left|\frac{\pi}{2}\right\rangle_a \left|\frac{\pi}{2}\right\rangle_b |0\rangle_c\right] \tag{3}$$

and where the outgoing photons reach the polarizers on the left before they reach the right. The traditional version of quantum mechanics predicts [19] a relative three-photon counting rate of

$$R_3/R_0 = 1/2(\cos\theta_a \cos\theta_b \sin\theta_c + \sin\theta_a \sin\theta_b \cos\theta_c)^2 \tag{4}$$

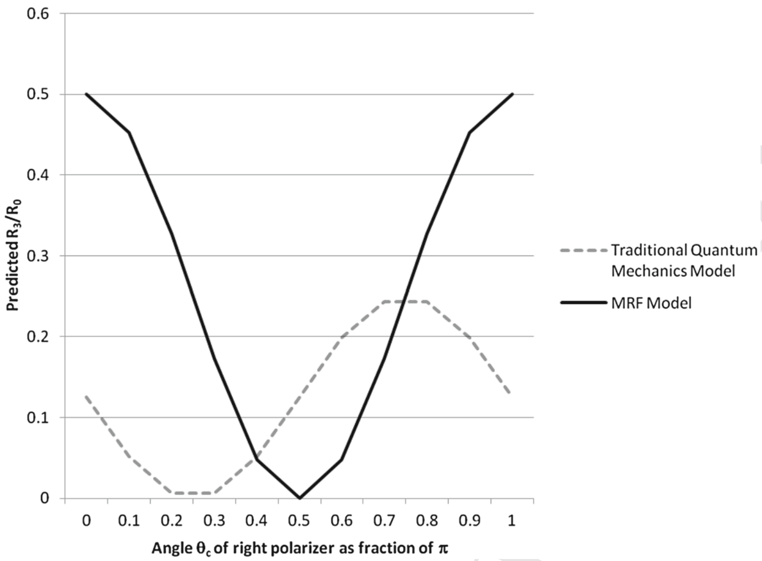

Fig. 4 Comparison of Traditional Quantum Mechanics Model and MRF Model

but the MRF models embodying time-symmetric physics [4,25] predic

$$R_3/R_0 = k\cos^2(\theta_c - \theta_a - \theta_b) \tag{5}$$

Conceptually, this experiment is as important as the Michelson–Morley experiment was, in constraining what kinds of models we should use in modeling every more complicated networks of entangled photons and polarizers. This entire family of experiments is so important as a starting point for AQC that it will be essential for verifying the circuit elements models many times over in the future. Presently, the numerical simulations of Eqs. 4 and 5 in Excel (see Fig. 4) demonstrate that there is no hidden trigonometric equivalence here across all choices of angles, and the competing models predict R_3/R_0 as shown in Fig. 4 for the case where $\theta_a = \pi/4$, $\theta_b = -\pi/4$, and the scaling factor k is chosen to be 0.5.

At the SPIE Conference in Baltimore, 2015, Dr. Yanhua Shih kindly shared in private communication the graphs of the results of his experiments on producing three entangled photons [28], using an extension of the new technology his team had recently used to replicate two-photon entanglement experiments [34]. The scatter graphs were clearly close to the theoretical prediction by MRF model in Fig. 4 in terms of period of the function, which is a half of a period predicted by traditional quantum mechanics model. This is very promising, since the two curves in Fig. 4 exhibit very different behavior. Further validation of MRF models for such systems calls for replication of these results with the triphotons generated by alternative methods, and the use of more complex types of MRF models which endogenously represent the beam splitters which are part of how Shih generated the simulated GHZ state. Shih has referred to the specific GHZ state given in Eq. 3 as the "asymmetric" GHZ state—the state which demonstrates triple entanglement, and not just coherence between the three photons. The high fidelity of qubits and CNOT operations with thermal light [22], and the existence of telltale zeroes in the R_2 and R_3 counting rates, suggests that his production of entanglement from thermal light offers great hope for the future.

In addition to the traditional ways of producing a GHZ source used in Zeilinger's group, and Shih's new thermal light approach, Fuli Li of Xian Jiaotong University has shown another way to combine squeezed light and beamsplitter filtering for ghost imaging, also presented at the Princeton-TAMU Workshop on Classical-Quantum Interface, Princeton, May 2015. This may offer yet another way to test the competing models here, but also calls for more complete modeling of the entire experimental apparatus by both modeling approaches.

If such new experiments continue to favor Eq. 5 over Eq. 4, as the preliminary results suggest, the implications are enormous, as will be discussed in Sect. 3.2. Traditional digital quantum computing becomes possible only with entangled states. It seems likely that the full power of AQC (exploiting phenomena specific to time-symmetric physics) requires exploiting triple entanglement, as in the GHZ state of Eq. 3, but not in the other symmetric or coherent state.

It would be very interesting to see whether use of triple entanglement in ghost imaging of remote objects in space would allow us to avoid the need for a second detector far away in space (as required for coincidence detection in the biphoton case [35]); if this works, it could be the first really practical application of triple entanglement. More concretely, the right channel could be directed to an astronomical object like the sun, as in standard ghost imaging, but with two channels on the left coincidence detection between these channels might be enough to filter out single photons in the ambient environment. Greater development of triply entangled sources is the key requirement for additional experiments on R_3/R_0, but would also be essential to ghost imaging with triple entanglement.

3.2 Time-symmetric physics: what the continuous triphoton experiment tests for

The continuous triphoton experiment, as specified above, offers a decisive test between two choices of models: the traditional quantum mechanics model versus MRF model, which presumes time-symmetric physics. It also provides information on what kinds of model we need to use in order to design and predict more complex systems using continuously tuned spin gates as we need for AQC. Time-symmetric physics, like special relativity itself, is a general concept which can be used to constrain and construct many different kinds of predictive models. Like special relativity, it can be used to constrain classical models based on partial differential equations (PDE) or canonical quantum field theories based on normal form Hamiltonians, or theories assuming a Lagrangian for use in the Feynman path approach. The central idea of time-symmetric physics is that we can and should go back to the idea of deriving probabilities of measurement by the same physical dynamics we assumed for the rest of reality, whether that dynamics be a Schrodinger equation or PDE or stochastic PDE or a four-dimensional generalization of quantum trajectory simulation of Carmichael [36]. In this view, the failure of classical physics to predict the Bell's Theorem experiments was not because of a failure of locality and realism, or limitations of PDE models, but because of an exogenous assumption not contained in the PDE themselves, the assumption of a particular type of time forwards causality. In real-life time series analysis, encountered in economics and finance [37], models based on that kind of time-forwards causality can often be written as:

$$\underline{\mathbf{S}}(t+\Delta t)=\underline{\mathbf{f}}\left(\underline{\mathbf{S}}\,(t)\,,\underline{\mathbf{e}}\,(t)\right) \tag{6}$$

where the random disturbances $\underline{\mathbf{e}}(t)$ obey the time-forwards causality assumption:

$$\langle \mathrm{S}\,(t)\,\mathrm{e}(\tau)\rangle = 0 \quad \text{whenever} \quad \tau > t \tag{7}$$

The predictions of "classical physics" assumed in the classic Bell's Theorems assume that this kind of causality assumption is appended to classical physics, to make actual predictions. The main result in [19] is that local realistic models can make the correct predictions if this appendage is removed, and if we adhere to the constraints of time-symmetric physics, which is already de facto in use in finance, economics and control theory [38]. As suggested in [16], we can find a way to answer this question by first going back to analyze the physical reasons why causality seems to flow forward in time so ineluctably in our region of space-time. It is all basically a matter of boundary conditions, of forwards flowing free energy derived from the Big Bang and the creation of our sun, which imposed boundary conditions at $-\infty$ in our neighborhood of the Universe.

A guideline is provided in [16] for how to build models consistent with time-symmetric physics, applicable both for macroscopic lumped parameter models and for models continuous in time and space. To be in conformance with time-symmetric methodology, the model of each object's dynamics in the experiment should be symmetric with respect to time, except when the object is a node where positive-time free energy is injected into the experiment, or, as a practical matter, when the backwards flow of energy is represented by the terms of higher order that make negligible contributions to the outcome probabilities. For the Bell Theorem experiments and triphoton experiments, the node of injection of energy is simply the source of entangled photons. These guidelines were used to construct the first local realistic models of the Bell experiments [16], which yielded the correct answer.

For now, the traditional quantum mechanics models and MRF models are detailed enough to predict a wide variety of experiments that can be done with many entangled photons. They are also the only models available now that have been proven to work in predicting actual outcomes of the experiment depicted in Fig. 1. The step before us in the roadmap of AQC is to further compare, refine and validate these lumped parameter models. However, for the long-term progress of the field, we need to go beyond pure lumped parameter models and create time-symmetric models in continuous time (and then continuous in space), as required to model hybrid systems which combine spin gates together with beam splitters, interference and other physical tools important to this technology. We have begun this process, first by developing continuous-time versions both of the standard operator projection model of the polarizer, and of the MRF models.

3.3 Possibilities for traditional and MRF models in continuous time and space–time

Both the traditional model and the MRF models treat the polarizers in these experiments as black box input–output systems. In most areas of quantum optics complex physical objects such as lasers, are modeled in a continuous time using density matrices and master equations. For the birefringent polarizer, such as calcite, the key action occurs at surface boundaries, so that a discrete-time approach actually can represent the physics of the process. But what about dichroic polarizers, also used in many Bell's Theorem experiments, where we know that polarization takes place in a continuous way, as photons propagate through the polarizers?

If the traditional model of the polarizer used in the successful quantum mechanical predictions of the Bell experiment [19,20,39] were to be verified in triphoton experiments, the following simple master equation can be used to describe how a photon propagates through a linear dichroic polarizer tuned to the angle θ_{p}, in the notation of [17] and [18]:

$$\dot{\rho} = ga\left(\theta_{\mathrm{p}} + \frac{\pi}{2}\right)\rho a^{+}\left(\theta_{\mathrm{p}} + \frac{\pi}{2}\right) \tag{8}$$

In this notation, the density operator ρ (rather than a wave function) describes our knowledge of the state of the incoming photons at any time, $a(\theta)$ is the absorption or annihilation operator for a photon of linear polarization θ, $a^{+}(\theta)$ is the corresponding creation operator, and θ_{p} and g are parameters of the polarizer. If the incoming photon is in a simple single-particle definite pure state, this master equation says that it enters a mixed state, described by a density matrix ρ which cannot be represented a simple pure state. In effect, it states that the outcome of a "measurement" by the polarizer is based on a stochastic projection to eigenvectors of the polarizer, exactly as in the calculations by Clauser and others [20,39]. I owe thanks to Rolf Binder of the University of Arizona for thoughts about how to model other types of spin gates, which helped inspire Eq. 8.

On the other hand, for the time-symmetric case, the continuous-time MRF model (CMRF) [33] of the dichroic polarizer is still based on probability theory. In general, for any physical process in the continuous-time case, each scenario or path $\{X(t)\}$ is a time series of a set X of discrete and continuous variables. The probability of any state $X(t)$ at time t can be calculated by Bayesian convolution of two partial probabilities, P^{+} and P^{-}, calculated by:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{Pr}^{+}(X) = -Z_{+}(t)\mathrm{Pr}^{+}(X) + \int G(X,Y)\mathrm{Pr}^{+}(Y)\mathrm{d}Y \tag{9}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{Pr}^{-}(X) = -Z_{-}(t)\mathrm{Pr}^{-}(X) + \int G(X,Y)\mathrm{Pr}^{-}(Y)\mathrm{d}Y \tag{10}$$

where Pr and Z variables are simple scalar real functions of time, while G is a local model of the object or system under study. Eqs. 9 and 10 are essentially just the continuous time version of Eqs. 1 and 2.

Equation 9 is solved forwards in time from the initial conditions, and Eq. 10 is solved in backwards time from the constraints on the final outcome imposed mainly by our final measurement apparatus. For the case of the dichroic polarizer, G(X,Y) is a simple and natural model of the local dynamics, given and used in [33], much more natural than the black box MRF models of a polarizer.

In cases like the Bell's Theorem and triphoton equation, where we are interested only in the final outcome, we do not actually need to use Eq. 10; we know Pr^{-} from our (very simple) model of what a detector does. The partition factors Z are calculated so as to normalize the partial probabilities Pr^{+} and Pr^{-} to add up to 1 at all times. (As in thermodynamics, the partition function looks like a simple scalar, but is of profound importance in understanding large-scale systems.)

The specific form of G proposed for the dichroic polarizer is based on relatively simple physics, yet the calculations show that it predicts the same kind of polarizer behavior which led to the prediction in Eq. 5 for the continuous triphoton experiment. This strongly suggests that any reasonable model based on time-symmetric physics, such as a more complex quantum many-worlds model [16], would yield the same prediction for that experiment.

When the general CMRF model is expressed in a more abstract way [33], it has striking similarity to the original Feynman path approach, from the time before Schwinger and others introduced the functional integration approach, when 'paths' were defined as a discrete set of continuous time-series, except that the continuous-time generalization of Eqs. 1 and 2 calculates probabilities rather than probability amplitudes. A major part of [33] is an incremental step-by-step development of CMRF as an extension of MRF, followed by the dichroic polarizer example, which some readers might prefer to go to directly.

The development of the CMRF models raises another very important question: if experiments should favor MRF models over traditional quantum mechanical models, how can that result be generalized to other areas of physics? There are three alternative ways one might proceed from there, to develop a full continuous-time continuous-space formulation consistent with the mass of experiments supporting the standard model of physics (with corrections to incorporate the empirical results from Cavity or Circuit QED (CQED), as discussed in [30]).

At the present time, the most promising approach for systems in electronics and photonics appears to be a variation of the stochastic path approach, which may be viewed as a four-dimensional generalization of Carmichael's Quantum Trajectory Simulation (QTS) method [39], which I would call Markov QED (MQED). In this approach, a scenario consists of a set of field values across space–time, and a countable set of events which may occur in principle at any point in space–time. Thus for example, the nodal probability Pr* of photon creation at any point in space–time is based on the interaction Hamiltonian H_{i}, with field values based on propagators of those fields transmitted from other events. An interesting benefit of this kind of model is that, as with Maxwell's Laws, one can use parameters or propagators describing propagation in a solid medium, such as Schwinger's Nonequilibrium Green's Functions [40], which are of great practical importance when making predictions of propagation through a solid object. Such propagators play an essential role in describing and predicting the interference phenomena. There still remain questions, which need to be investigated, such as the role of two-photon lines and three-photon lines in Feynman diagrams, as necessary to account for empirical phenomena of central importance in quantum optics [20].

4 Conclusions and further developments

The full development of AQC technology would require several new streams of research in parallel, in addition to the worthy, complementary research on digital quantum computing, on traditional continuous-value quantum computing and on new approaches to decoherence, dissipation and disentanglement beyond the scope of this paper. These new streams should include new experiments to validate and extend either of the lumped parameter models of polarizers and spin gates discussed here (or conceivably developed yet another dissipative modeling of photon density operators), in a progression form triple entanglement to more complex systems, similar to the route already followed by Zeilinger for digital quantum computing. New types of ghost imaging of objects, such as the Sun, could provide a good early niche application for continuous triple entanglement. Another new stream of research involves new theoretical work, leading to further supporting experiments, in order to develop more complex continuous-time models for circuits combining not only spin gates but beam splitters and interference and so on, endogenous to the models. A third stream would include theoretical work to develop more comprehensive models in continuous space as well as time, consistent with what is learned from the first two directions of research. Finally, a fourth parallel stream would attempt to replicate what is achieved in photonics through similar work on other types of particles, such as excitons, polaritons and plasmons, starting with replications of Bell's Theorem experiments, and ultimately perhaps allowing smaller feature size and greater density in computing than is possible with photons proper.

Even the Markov QED type of model discussed as a goal in Sect. 3.3 would not be a complete model of what underlies electricity and light, because it does not account for interactions with other forces. For those who seek to find the "law of everything," it is still just a phenomenological model of stochastic emergent phenomena, like QED itself—but QED is already powerful enough to allow new types of quantum computing and intelligent systems beyond what most people can imagine yet today.

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